♦ A Two-Stage Market Model for Microgrid Power Transactions via Aggregators

Hongseok Kim and Marina Thottan

In this paper, we propose a market model where microgrids sell their surplus power to a utility via aggregators. This is a scalable model where a utility does not directly interact with a large number of microgrids. Thus, aggregators collect power from microgrids and resell it to the utility. From the microgrids' perspective, aggregators are buyers. From the utility's perspective, aggregators are sellers. In this context, based on the two-stage Stackelberg game, we show how to achieve efficient market equilibrium using the tatonnement process and supply function bidding. We also show that the participation of aggregators may significantly affect the market depending on the supply elasticity of microgrids, which in turn depends on the cost structure of microgrids. For example, when the cost function of microgrids is roughly linear, the aggregators may not make a profit. However, if the cost function of microgrids has a higher order term, aggregators may accumulate a large profit, which potentially raises the issue of the regulator's role in the market. © 2011 Alcatel-Lucent.

Introduction

Smart grids will be comprised of many different types of power generators. Unlike traditional power generation methods based on a small number of bulk power generators using hydro, nuclear, or coal, in the smart grid, it is expected that there will be a large number of distributed generators with small scale generation capacity including solar photovoltaic, wind farm, fuel cell, and micro combined heat and power (CHP) providers. These new energy sources are usually called distributed energy resources (DERs). Thanks to the recent development of DERs, *microgrids* are emerging as a new power generation method. A microgrid is a power system having at least one DER to serve its internal load. It performs an intentional islanding operation upon a power disturbance in the electrical distribution system; however, in a normal mode of operation, microgrids are connected to the grid and can import/export power from/to the power grid [2]. In this paper we consider a market model where microgrids have surplus power and thus incentive to participate in the power market. Our goal is to capture the market behavior when there are a large number of microgrids, each with small generation capacity. Specifically, the direct participation of individual microgrids in the retail market may not be scalable since a utility faces the problem of dealing with a large number of small power sources [2]. To address this scalability problem, we introduce the concept of

Bell Labs Technical Journal 16(3), 101–108 (2011) © 2011 Alcatel-Lucent. Published by Wiley Periodicals, Inc.
Published online in Wiley Online Library (wileyonlinelibrary.com) • DOI: 10.1002/bltj.20524

aggregators. An aggregator is an entity which collects power generated from microgrids and resells it to the utility. Power aggregation is also beneficial from a communications perspective because otherwise it would be necessary to deploy communication facilities for handling the huge amount of transaction information between the utility and microgrids. Aggregators profit by setting a margin between the buying and selling price. Our market model explores the market behavior in the presence of aggregators. We use the profit maximization framework where market participants, i.e., utility, aggregators and microgrids each make decisions to maximize their own profits [3].

Panel 1. Abbreviations, Acronyms, and Terms

CHP—Combined heat and power
DER—Distributed energy resource
GERI—Gachon Energy Research Institute
ISO—Independent system operators
PV—Photovoltaic
RTO—Regional transmission organization
V2G—Vehicle-to-grid

Microgrid Power Transaction via Aggregators

In this section we describe a two-stage market model that captures the impact of the aggregators on the microgrid power transaction.

System Model

Suppose that during a period of peak usage on a hot summer day, the utility expects a power deficit and needs to purchase more power to meet demand. In this case, microgrids can quickly supply power to the grid at a lower price than the spot market price because microgrids have very responsive power generation sources such as micro turbines, fuel cells, CHP, and energy storage [2]. From the definition of a microgrid, an electric vehicle can be considered to be a very small microgrid and serve as a power source for the power grid. (This arrangement is often referred to as vehicle-to-grid (V2G).) In addition, a microgrid can also be a very large system having different types of power sources. Microgrids are also good for providing ancillary services such as reactive power and voltage control, and supply of reserves (frequency responsive spinning reserve, supplemental reserve, and backup supply) [2]. For simplicity, however, and without loss of generality, we only consider real power. We also assume that power networks are bidirectional and lossless with infinite capacity [5, 6]. We will therefore ignore the complexities introduced by the transmission and distribution networks and concentrate on the transaction of electrical power. However, in reality, transmission constraints and losses in the network connecting generators and loads can introduce gross distortion in the market [5].

We consider a two-stage market as shown in **Figure 1** where the microgrids and the utility transact power via aggregators. Independent system operators (ISO) or regional transmission organizations (RTOs) can attend to the transaction instead of the utility, but for simplicity we consider the utility. The first stage market is between the utility and the aggregators. The second stage market is between the aggregator and the microgrids. This market can be modeled by a two-stage Stackelberg game. At the first stage market, the utility is a leader and the aggregators are follows; the utility announces the first stage price for power procurement, and then the aggregators adapt their decisions (determine the supply quantity) to maximize their own profits. Because the aggregators do not have power generation sources, however, they need to purchase power from microgrids at the second stage market. Hence, the aggregators announce the second stage prices as leaders, and microgrids decide the supply quantities to maximize their own profits as followers. We assume both the first stage market and the secondary market are competitive, and thus aggregators and microgrids behave as price-takers [7]. As can be seen in Figure 1, each microgrid is associated with one aggregator.

We consider the problem where the utility wants to procure D amount of power from the aggregators at a minimum purchasing price. Let $j \in A$ denote the index for the aggregator, and $i \in M_j$ denote the index for the microgrids associated with the aggregator j. Let x_j denote the amount of power the aggregator j supplies to the utility, and $c_j(x_j)$ denote the associated

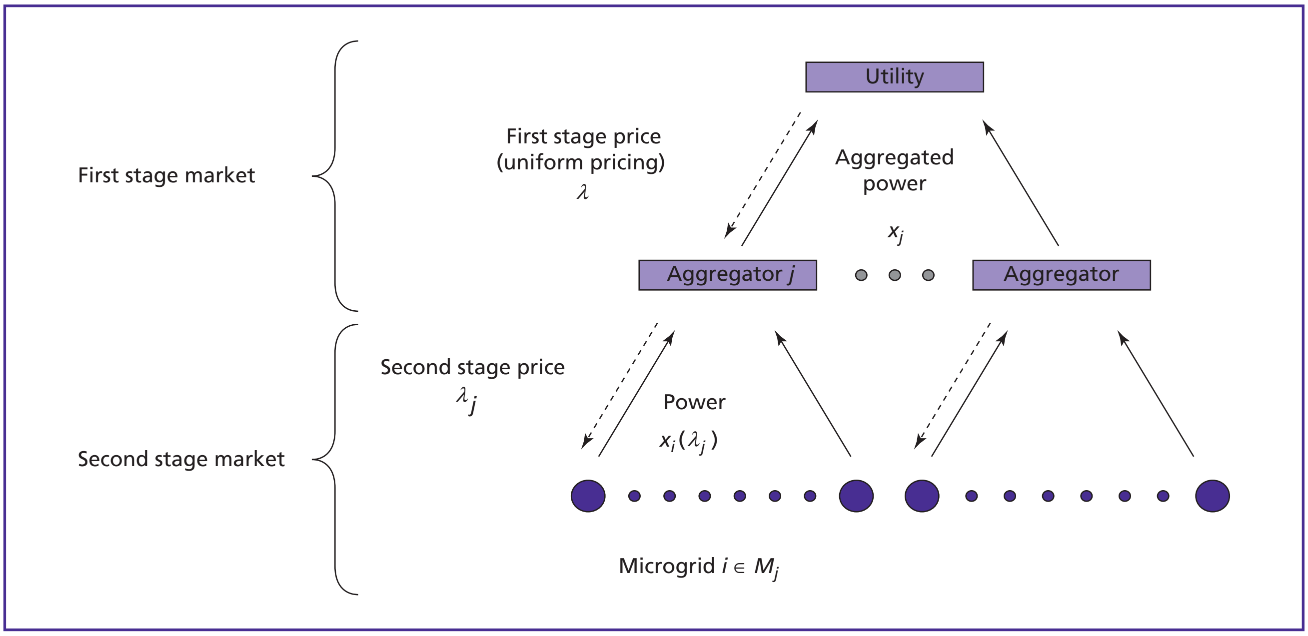

Figure 1.
Two-stage market model for power transaction via aggregators.

aggregation cost, i.e., the cost of procuring x_j from microgrids $i \in M_j$. We assume that $c_j(x_j)$ is strictly convex, monotone increasing, differentiable, and zero at $x_j = 0$. In a two-stage Stackelberg game, as can be seen in Figure 1, the utility announces the first stage price λ, and then, aggregators $j \in A$ respond x_j to the utility. Then, to collect x_j from the microgrids, aggregator j announces the second stage price λ_j, and in turn each microgrid $i \in M_j$ responds $x_i(\lambda_j)$ to aggregator j.

First Stage Market

From the utility's perspective, the power procurement process is an optimization problem given by Utility:

$$\begin{aligned} &\text{minimize} && \lambda \\ &\text{subject to} && \sum_{j \in A} x_j = D \qquad (1) \\ &\text{variable} && \lambda \end{aligned}$$

where x_j is a function of λ. One can show that solving the utility's optimization problem, equation 1, is equivalent to solving the cost minimization problem of the aggregators [2],

Aggregators' cost:

$$\begin{aligned} &\text{minimize} && \sum_{j \in A} c_j(x_j) \\ &\text{subject to} && \sum_{j \in A} x_j = D \qquad (2) \\ &\text{variable} && \{x_j \mid j \in A\}. \end{aligned}$$

Then, λ serves as the Lagrange multiplier of equation 2 such as

$$\begin{aligned} L &= \sum_{j \in A} c_j(x_j) - \lambda\left(\sum_{j \in A} x_j - D\right) \\ &= \sum_{j \in A} (c_j(x_j) - \lambda x_j) + \lambda D. \end{aligned} \qquad (3)$$

Minimizing L can be done by the method called *dual decomposition* where L is decomposed into $|A|$ number of sub-problems [3]. Then, equation 3 is minimized by an iterative market mechanism in a distributed way. This iteration is called *tatonnement process* [4, 6]. Specifically, at the k-th iteration, the utility announces the price λ^k and aggregator j responds x_j^k, which is the optimal solution of the following optimization problem,

Aggregator $j \in A$:

$$x_j^k = \arg\max_{x_j} \lambda^k x_j - c_j(x_j). \tag{4}$$

After the utility collects x_j^k from all aggregators, it updates the next price using, for example, the *dual gradient method* given by $\lambda^{k+1} = \left[\lambda^k - \delta^k\left(\sum_{j \in A} x_j^k - D\right)\right]^+$ where δ^k serves as the price-updating step size. One can show that if the step size is chosen small enough, this iteration converges to the market equilibrium with the market clearing price denoted by λ^* [6]. Then, the market is called *efficient* because, at the equilibrium, the utility minimizes its power procurement cost while each aggregator also maximizes its profit [3]. Our work in [4] shows that the *bisection method* and *Illinois method* exhibit much faster convergence speed, in fact, at least exponentially fast compared to the dual gradient method, without the concern of determining the proper step size for convergence. Furthermore, the bisection method and Illinois method are highly scalable, and the number of iterations is given by O(1), i.e., does not grow up as $|A|$ grows. The dual gradient method can also achieve O(1) iterations, but only if the proper step size is known a priori.

Second Stage Market

During the iteration, given λ^k, the aggregator j needs to solve equation 4, the second stage market problem, to communicate x_j^k to the utility. Unlike the iterative bargaining process in the first stage market, supply function bidding is considered at the second stage market for two reasons:

1. An iterative solution for both the first stage market and the second stage market may require an excessively large number of iterations because of the cascaded iteration.
2. The microgrid is usually assumed to have an intelligent device that knows its power generation cost.

Hence, the microgrid $i \in M_j$ knows its optimal supply function, which is a function of the second stage price λ_j and given by the solution of the following sub-problem.

Microgrid $i \in M_j$:

$$\begin{array}{ll} \text{maximize} & \lambda_j x_i - c_i(x_i) \\ \text{variable} & x_i \end{array} \tag{5}$$

where $c_i(x_i)$ is a cost function of microgrid $i \in M_j$ in exporting x_i amount of power. We assume that $c_i(x_i)$ is *strictly* convex, monotone increasing, differentiable, and $c_i(0) = 0$ (i.e., we consider the short run only). From the first order condition of equation 5, the supply function of the microgrid is given by the inverse of the marginal cost, i.e., $x_i(\lambda_j) = (c_i')^{-1}(\lambda_j)$ [7]. Then, each microgrid bids its supply function to the aggregator so that the aggregator j has the complete knowledge of the pair of the information, the second stage price, and the associated generation quantity, i.e., $x_j(\lambda_j) = \sum_{i \in M_j} x_i(\lambda_j)$. This supply function bidding indeed maximizes the microgrids' profit because the second stage market is competitive. However, if the market is not competitive, i.e., an oligopoly, microgrids may submit a different type of supply function to exercise market power. The oligopoly market will lead to the study of *supply function equilibrium* [1], which is out of scope of this paper.

Once the aggregator j has $x_j(\lambda_j)$, the cost function of the aggregator j is given by $c_j(x_j) = \lambda_j x_j(\lambda_j)$. We assume that the aggregation cost only comes from the logical power purchasing cost and does not include the costs that may be incurred for the physical power transaction, e.g., maintaining a distribution network. Given λ^k, since x_j is a function of λ_j, the aggregator j can solve the optimization problem of equation 4 with respect to λ_j (instead of x_j) and thus determine

$$\lambda_j^k = \arg\max_{\lambda_j} \lambda^k x_j - c_j(x_j(\lambda_j)),$$

i.e., the optimal second stage price that maximizes the profit of aggregator j given λ^k. Then, the optimal x_j^k is given by $x_j^k = x_j(\lambda_j^k)$. This second stage computation needs to be done whenever λ^k is updated in the first stage market. Note that λ_j^k serves as the optimal Lagrange multiplier of the cost minimization problem in procuring x_j^k from the microgrids, i.e.,

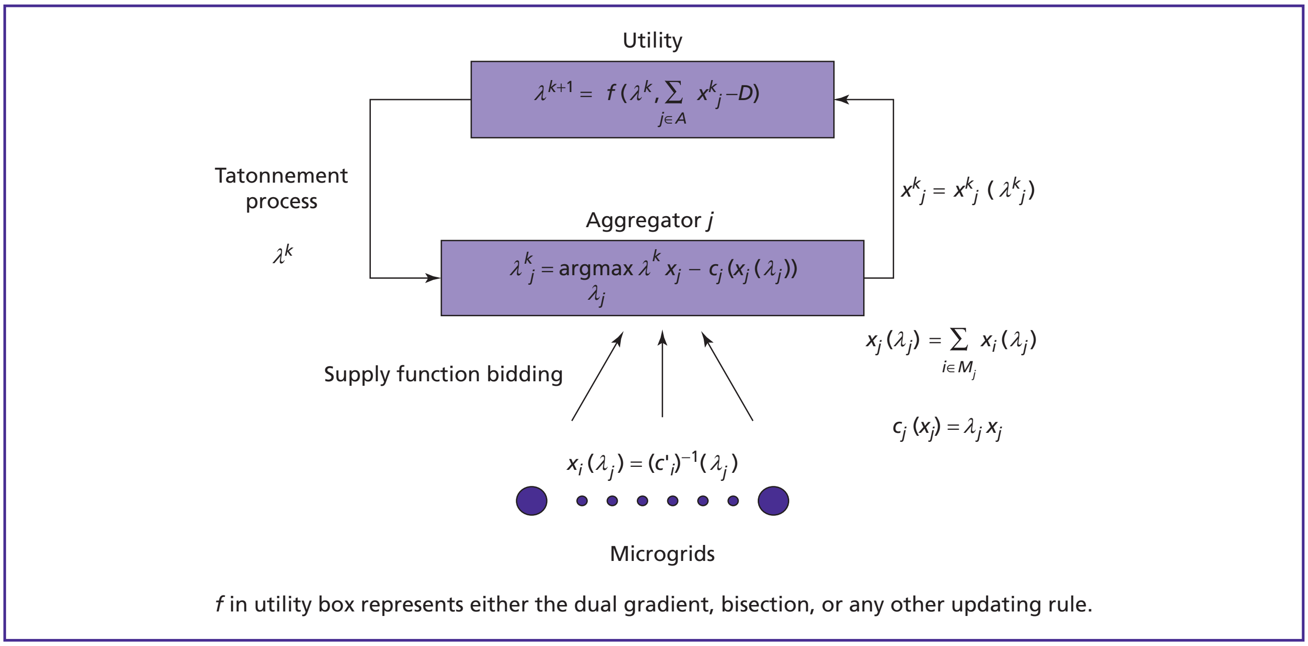

Figure 2.
Iteration process based on tatonnement process and supply function bidding.

Second stage market:

$$\begin{aligned} &\text{minimize} && \sum_{i\in M_j} c_i(x_i) && (6)\\ &\text{subject to} && \sum_{i\in M_j} x_i = x_j^k, \\ &\text{variable} && \{x_i \mid i \in M_j\}. \end{aligned}$$

Figure 2 summarizes the bargaining process based on the tatonnement process (first stage) and supply function bidding (second stage), which converges to the market equilibrium where all market participants maximize their profits. At equilibrium, if any of the market participants fail in fulfilling the commitments, a penalty fee is charged based on a previously-negotiated contract. The function f in the utility implies that updating the first stage price can be either based on the dual gradient, bisection, Illinois or any other methods. One might want to consider the supply function bidding at the first stage market as well by submitting x_j as a function of λ, but note that the aggregator only has $x_j(\lambda_j)$.

The Profits of Aggregators

Now we are interested in the profits of the aggregators at market equilibrium. The profit of the aggregator is determined from the difference between the first stage and the second stage price at market equilibrium.

Proposition 1. At equilibrium, the optimal first stage price λ^* and the second stage price λ_j^* have the following relationship, $\lambda_j^* = \dfrac{\lambda^*}{1 + 1/e_j^*}$ where $e_j^* = e_j(\lambda_j^*)$ and $e_j(\lambda_j) = \dfrac{dx_j \lambda_j}{d\lambda_j x_j}$ is the aggregated supply elasticity of microgrids $i \in M_j$, i.e., the normalized metric of the power supply change upon price change [7].

Proof. Since $x_j(\lambda_j)$ is a monotone increasing function, x_j and λ_j have one-to-one correspondence, and the inverse function $\lambda_j = \lambda_j(x_j)$ exists. For any given λ, equation 4 is maximized when the first order condition is met, i.e., $\dfrac{d}{dx_j}(\lambda x_j - \lambda_j x_j) = \lambda - \dfrac{d\lambda_j}{dx_j} x_j - \lambda_j = 0$, and thus we have the following equation $\lambda = \dfrac{d\lambda_j}{dx_j}\dfrac{x_j}{\lambda_j}\lambda_j + \lambda_j = \left(\dfrac{1}{e_j} + 1\right)\lambda_j$, which also holds at equilibrium.

From proposition 1, we see that the profit of the aggregator is given by $\lambda^* x_j^* - \lambda_j^* x_j^* = \dfrac{\lambda_j^* x_j^*}{e_j^*}$, and the ratio of profit over the cost is $1/e_j^*$. Hence, the profit of the aggregators heavily depends on the aggregated supply elasticity of $x_j(\lambda_j)$ which in turn depends on the cost function of microgrids. Typically, the short run cost function of the bulk power generator is modeled by a quadratic function, e.g., $c_i(x_i) = a_i x_i^2 + b_i x_i$ where the ratio of a_i and b_i depends on the generation method [1]. In the case of microgrids, the cost function depends on the various micro-generation technologies such as fuel cells, micro turbines, CHP, wind farms, and solar photovoltaic (PV). Hence, to abstract the cost function and have better intuition, we consider the following exemplary form, $c_i(x_i) = \sigma_i x_i^{1+\alpha_i}/(1 + \alpha_i)$ where α_i determines the order of the cost increase while σ_i captures the cost variation [4]. The supply function of the microgrid i is then $x_i(\lambda_j) = (\lambda_j/\sigma_i)^{1/\alpha_i}$. One can show that the supply elasticity of $x_i(\lambda_j)$ (i.e., of individual microgrid $i \in M_j$) is given by $\frac{dx_i}{d\lambda_j}\frac{\lambda_j}{x_i} = \frac{1}{\alpha_i}$ irrespective of σ_i and λ_j. Then, if $\alpha_i = \alpha$ for all $i \in M_j$, we have $e_j = 1/\alpha$, i.e., a *constant* aggregated supply elasticity. This simple example gives us the following intuition; when the cost functions of microgrids are close to linear (i.e., when α is close to 0), the profit of the aggregator is also close to zero. However, as the cost functions of the microgrids exhibit more curvature (i.e., when α increases), the aggregated supply elasticity decreases, and the aggregator's profit increases. For example, if the cost function is purely quadratic, i.e., $\alpha = 1$, then, $e_j = 1$, and the ratio of profit over cost becomes significant, i.e., 100 percent; if $\alpha = 2$, then $e_j = 1/2$, and the ratio becomes 200 percent. Hence, the lower the supply elasticity of the microgrids, the higher profit aggregators can have.

Discussion of the Two-Stage Market and the Role of Regulator

The above observation suggests several possible market scenarios and the role of the regulator. We first consider the case when the cost function of the microgrid is close to linear and thus the aggregators achieve very little profit. One might think this is desirable because almost zero profit for the aggregators implies a low procurement cost for the utility, and thus eventually a low electricity price for the end customer. However, this will discourage aggregators from participating in the market. Then, without aggregators, microgrids will have to negotiate directly with utilities to sell their surplus power; this will fractionate the overall microgrid supply among individual microgrids, and hence the utility will face a scalability issue in terms of cost. As a consequence, a market with aggregators may not exist, which corresponds to case 1 in **Table I**. Microgrids with large generation capacity, however, may directly transact power with utilities, as in case 2. The case where regulators have a role is when cost functions have a dominant a higher order term, and thus aggregators can reap large profits. Since this leads directly to higher prices for consumer electricity, regulators should be involved to limit aggregator profit-taking, as in case 3. Finally, when microgrids have large general capacity and cost functions also have a dominant higher order term, as shown in case 4, aggregators may or may not be necessary, depending on the number of microgrids.

Table I. Four possible market scenarios depending on the cost function and the generation capacity of the microgrid.

	Small-scale microgrid	Large-scale microgrid
More like linear cost function	**Case 1** A market with aggregators may not exist, and microgrids may not transact power with the utility.	**Case 2** Aggregators would not participate in the market, but large-scale microgrids can directly transact power with the utility.
Cost function with a higher order term	**Case 3** Aggregators may make too much profit, and then regulation may become necessary.	**Case 4** Transaction is either direct or via aggregator, depending on the number of microgrids.

Discussion of the Linear Cost Function

It should be noted that our mathematical model assumes a strictly convex and monotonically increasing cost function for the microgrid, which excludes the case of linear cost function. This is because the supply function of the microgrid is not defined in the case of linear cost function and thus the analytic approach taken in this paper cannot hold.

Conclusion

In this paper we analyzed a market model where microgrids transact power with a utility via aggregators. We modeled this market as a two-stage Stackelberg game and analyzed the behaviors using a profit maximization framework. To achieve efficient market equilibrium, we proposed a tatonnement process at the first stage market and supply function bidding at the second stage market. Finally, we found that the aggregator's profit depends largely on aggregated supply elasticity, which again depends on the cost function of the microgrids. Interestingly, when the cost function of the microgrids is close to linear, aggregator profit is expected to be marginal. However, aggregator profit can be significant if the cost function has a higher order term, which suggests a role for the regulator. Determining a realistic cost function for the microgrid environment remains an open problem for future study.

Acknowledgements

This work has been conducted under a joint research program between Alcatel-Lucent Bell Labs, Murray Hill, New Jersey and the Gachon Energy Research Institute (GERI), with support from Kyungwon University of Korea.

References

[1] R. Baldick, "Electricity Market Equilibrium Models: The Effect of Parameterization," IEEE Trans. Power Syst., 17:4 (2002), 1170–1176.

[2] S. Chowdhury, S. P. Chowdhury, and P. Crossley, Microgrids and Active Distribution Networks, Institution of Engineering and Technology, Stevenage, UK, 2009.

[3] F. Kelly, "Charging and Rate Control for Elastic Traffic," Eur. Trans. Telecommun., 8:1 (1997), 33–37.

[4] H. Kim, Y.-J. Kim, K. Yang, and M. Thottan, "Cloud-Based Demand Response for Smart Grid: Architecture and Distributed Algorithms," Proc. 2nd IEEE Internat. Conf. on Smart Grid Commun. (SmartGridComm '11) (Brussels, Bel., 2011).

[5] D. S. Kirschen and G. Strbac, Fundamentals of Power System Economics, John Wiley & Sons, Chichester, West Sussex, Eng., Hoboken, NJ, 2004.

[6] A. Papavasiliou, H. Hindi, and D. Greene, "Market-Based Control Mechanisms for Electric Power Demand Response," Proc. 49th IEEE Conf. on Decision and Control (CDC '10) (Atlanta, GA, 2010), pp. 1891–1898.

[7] H. R. Varian, Microeconomic Analysis, Norton, New York, 1992.

(Manuscript approved June 2011)

HONGSEOK KIM was a member of technical staff in the Alcatel-Lucent Bell Labs Network Performance and Reliability Department in Murray Hill, New Jersey, USA from 2010 to 2011. He is now an Assistant Professor at the Department of Electronic Engineering, Sogang University, Seoul, Korea. He received his B.S. and M.S. degree in electrical engineering from Seoul National University, Seoul, Korea, and his Ph.D. degree in electrical and computer engineering at the University of Texas at Austin. He was a post-doctoral research associate in the Department of Electrical Engineering at Princeton University, Princeton, New Jersey. He was also a member of technical staff at Korea Telecom Network Laboratory for five years prior to his tenure at Alcatel-Lucent, where he participated in Full Service Access Network (FSAN) and ITU-T SG16 FS-VDSL standardization efforts. His research interests include smart grid, demand response, information middleware, network resource allocation, optimization, network economics, and cross layer design of wireless communication systems. Dr. Kim has authored more than 30 papers in IEEE and ACM journals.

MARINA THOTTAN is director of the Mission-Critical Communications and Networking Group at Alcatel-Lucent Bell Labs in Murray Hill, New Jersey, USA. She has contributed to a wide variety of projects in diverse subject areas including online gaming, content distribution, Voice over IP (VoIP), routing protocols, data over optical networks, high speed router design, network management and anomaly detection. Most recently she has been leading work on smart grid communication networks. She holds a Ph.D. degree in electrical and computer engineering from Rensselaer Polytechnic Institute, Troy, New York. Dr. Thottan has published over 40 papers in scientific journals, book chapters, and refereed conferences. She is a member of both the IEEE and ACM. ♦