Pricing-based Spectrum Access Control in Cognitive Radio Networks with Random Access

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Abstract—Market-based mechanisms are promising approaches for spectrum access in cognitive radio networks. In this paper, we focus on two market models, one with a monopoly primary user (PU) market and the other with a multiple PU market, where each PU sells its temporarily unused spectrum to secondary users (SUs). We propose a pricing-based spectrum trading mechanism that enables SUs to contend for channel usage by random access, in a distributed manner, which in turn mitigates the complexity and time overhead associated with centralized scheduling. For the monopoly PU market model, we start with SUs contending via slotted Aloha. The revenue maximization problems here are nonconvex. Observing that the global optimal pricing strategy is Pareto optimal, we first characterize the Pareto optimal region, and then obtain a Pareto optimal solution that maximizes the SUs' throughput subject to the SUs' budget constraints. To mitigate the spectrum underutilization due to "price of contention," we revisit the problem where SUs content via CSMA. Spectrum utilization is enhanced, resulting in higher revenue. When the PU's unused spectrum is a control parameter, we study further the tradeoff between the PU's utility and its revenue.

Next, for the multiple PU market model, we cast the competition among PUs as a three-stage Stackelberg game, where each SU selects a PU's channel to maximize its throughput. We characterize the Stackelberg equilibria, in terms of access prices and the spectrum offered to SUs. Our findings reveal that the number of equilibria has a threshold structure: if the number of PUs is greater than the threshold, there exist infinitely many equilibria; otherwise, there exists a unique equilibrium, where the access prices and spectrum opportunities are determined by the budgets/elasticity of SUs and the utility level of PUs.

I. INTRODUCTION

Cognitive radio (CR) is expected to capture temporal and spatial "spectrum holes" in the spectrum white space, and to enable spectrum sharing for secondary users (SUs). One challenge is that SUs should be able to discover spectrum holes and access them efficiently and fairly, without causing interference to the primary users (PUs), especially when the demand for available spectrum nearly outstrips the supply.

Market-based mechanisms have been explored as an important approach for spectrum access. Among these mechanisms, auction-based spectrum access mechanisms have been extensively studied for different objectives, e.g., incentive compatibility [1]–[4], spectrum reuse [1], [2], [5], [6], auctioneer's revenue maximization [2], social welfare maximization [6], power allocation schemes for the SUs with interference protection for the PU [7], etc. These works mainly focus on

on-demand auctions where each SU requests spectrum based on its traffic demand, and the time overhead can be significant in the auction procedure. Compared with auction-based spectrum access mechanisms, pricing-based spectrum access mechanisms may incur lower time overhead (see [8]–[14] and the references herein). Among these approaches, [8] studied pricing policies for a PU to sell unused spectrum to multiple SUs by using stochastic dynamic programming. [9], [10] considered competition among multiple PUs that sell spectrum to multiple SUs, whereas [11] focused on competition among multiple SUs to access the PU's channels. [12] considered spectrum trading across multiple PUs and multiple SUs, and formulated the price competition among PUs and spectrum selection of SUs as a noncooperative game. [13] studied the investment and pricing decisions of a network operator under spectrum supply uncertainty by modeling the problem as a multi-stage Stackelberg game. These existing works assume *orthogonal* multiple access among SUs, either in time or frequency domain, where a central controller is needed to handle SUs' admission control, to calculate the prices, and to charge the SUs. The computational complexity for dynamic spectrum access and the need of centralized controllers can often be overwhelming. To address these problems, recent work [14] proposed a two-tier market model based on the decentralized bargain theory, where the spectrum is traded from a PU to multiple SUs on a larger time scale, and then redistributed among SUs on a smaller time scale. However, [14] only considered a market with single PU and multiple homogeneous SUs.

As a less-studied alternative, random access can be employed for distributed spectrum access to mitigate the overwhelming requirements. With this motivation, we focus on the impact of pricing-based random access on dynamic spectrum access. In particular, we study the behaviors of PUs and SUs in different spectrum trading markets based on random access for two models: one with a monopoly PU market and the other with a multiple PU market.

In the monopoly PU market model, when the PU's unused spectrum is fixed, we study pricing-based spectrum access control for slotted Aloha, by searching for an optimal pricing strategy to maximize the PU's revenue. Due to nonconvexity of the problem, it is often hard to find the global optimum. Instead, we first characterize the Pareto optimal
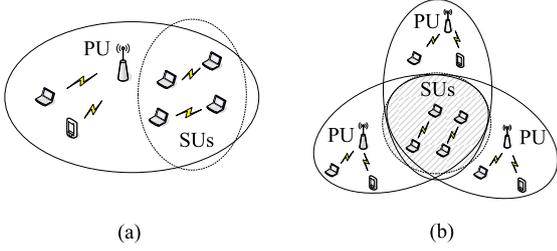
Fig. 1: (a) Monopoly PU Market (b) Multiple PU Market.

region associated with the SUs' throughput, based on the key observation that the global optimal strategy is also Pareto optimal. Then, by maximizing the SUs' throughput subject to the budget constraints, we provide a Pareto optimal solution that takes into account both the spectrum utilization and the SUs' budgets. This Pareto optimal solution exhibits a decentralized structure, i.e., the Pareto optimal pricing strategy and access probabilities can be computed by the PU and the SUs locally. We then develop a distributed pricing-based spectrum access control algorithm accordingly. To mitigate the spectrum underutilization due to "price of contention," we next turn to dynamic spectrum access using CSMA. Improvements in spectrum utilization and revenue to PU are quantified. We also consider the PU's unused spectrum as a control parameter, when the PU can flexibly allocate the spectrum to its ongoing transmissions. This enables us to tune the tradeoff between the PU's utility and its revenue.

In the multiple PU market model, we treat the competition among PUs as a three-stage Stackelberg game, where each PU seeks to maximize its net utility. We explore optimal strategies to adapt the prices and offered spectrum for each PU. We show that the equilibria of the game exist and depend on the system parameters such as the number of PUs, SUs' budgets and demand elasticity. Our results reveal that the number of equilibria exhibits a threshold phenomenon: if the number of PUs is greater than the threshold, there exist infinitely many equilibria, where the equilibrium price reduces to the minimum price that may cut down PUs' profit; otherwise, there exists a unique equilibrium. We further provide an iterative algorithm to find the equilibrium.

The rest of the paper is organized as follows. In Section II, we study the monopoly PU market, and present the Pareto optimal pricing strategy for the PU's revenue maximization problem. We also characterize the tradeoff between the PU's utility and its revenue. We study the competition among PUs in the multiple PU market in Section III, where we cast the competition among PUs as a three-stage Stackelberg game, and analyze the equilibria of the game. Finally, we conclude the paper in Section IV. Due to space limitation, the details for the proofs are omitted and can be found in [20].

II. MONOPOLY PU MARKET

A. System Model

We first consider a monopoly PU market with a set of SUs, denoted by $\mathcal{M}$. The PU sells the available spectrum opportunity c in each period[1], in terms of time slots in a slotted wireless system, to SUs who are willing to buy the spectrum opportunities (Figure 1(a)). In this case, the PU broadcasts the availability of spectrum opportunities and the prices to access the spectrum to the SUs. If a SU decides to buy the spectrum opportunity, it sends a request message to the PU.

We study two cases: 1) the spectrum opportunity c is fixed, where the PU desires to find the optimal pricing strategy (the optimal usage price and flat price) to maximize its revenue; 2) the spectrum opportunity c is a control parameter that the PU can use to balance its own utility and revenue. In both cases, each SU seeks to set its demand that maximizes its payoff, given the spectrum opportunity c, the usage price p_i, and the flat price g_i.

B. The Case with Fixed Spectrum Opportunity

We first study the case where the spectrum opportunity c is fixed. We will show that the Pareto optimal usage price is the same for all SUs, and is uniquely determined by the demand of each SU and c. We begin with the channel access model for SUs, assuming a linear pricing strategy, i.e., the PU charges each SU a flat price and a usage price proportional to its *successful* transmissions.

1) Slotted Aloha Model for SUs' Channel Access

Each SU first carries out spectrum sensing to detect the PU's activity. When the sensing result reveals that the PU is idle, SUs will contend for channel access by random access. As in the standard slotted Aloha model, we assume that all SUs are within the contention ranges of the others and all transmissions are synchronized in each slot. We assume that SUs always have enough packets to transmit and traffic demands of SUs are elastic. Denote by z_i the transmission probability of the ith SU. The probability that the ith SU's packet is successfully received is $s_i = z_i \prod_{j \neq i}(1 - z_j)$. The expected number of successfully transmitted packets of the ith SU in one period can be written as $\hat{s}_i = c s_i$, where $\hat{s}_i \in [0, c]$. Accordingly, the ith SU receives a utility in one period equal to $U_i(\hat{s}_i)$, where $U_i(.)$ denotes the utility function of the ith SU. The optimal demand $\hat{s}_i^*$ is the solution to the following optimization problem:

$$\begin{array}{ll} \text{maximize} & U_i(\hat{s}_i) - (p_i \hat{s}_i + g_i) \\ \text{subject to} & 0 \leq \hat{s}_i \leq c \qquad\qquad (1) \\ \text{variable} & \{\hat{s}_i\}. \end{array}$$

As is standard, we define the demand function that captures the successful transmissions $\hat{s}_i$ given the price p_i as

$$d_i(p_i) = \begin{cases} U_i'^{-1}(p_i) & \text{if } g_i \leq U_i(\hat{s}_i) - p_i \hat{s}_i, \\ 0 & \text{otherwise.} \end{cases} \qquad (2)$$

Assuming α-fair utility functions, the utility and the demand

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[1]The period refers to a time frame that the PU sells its unused part, denoted by c.

functions of each SU can be written, respectively, as:

$$U_i(\hat{s}_i) = \begin{cases} \frac{\sigma_i}{1-\alpha}(\hat{s}_i)^{1-\alpha} & 0 \le \alpha < 1, \\ \sigma_i \log(\hat{s}_i) & \alpha = 1, \end{cases} \quad (3)$$

$$d_i(p_i) = \begin{cases} \sigma_i^{\frac{1}{\alpha}} p_i^{-\frac{1}{\alpha}} & \text{if } g_i \le U_i(\hat{s}_i) - p_i \hat{s}_i, \\ 0 & \text{otherwise}, \end{cases} \quad (4)$$

where σ_i, the multiplicative constant in the α-fair utility function, denotes the utility level[2] of the ith SU, which reflects the *budget* of the ith SU. Note that we mainly consider the case of $0 < \alpha < 1$ because $\frac{1}{\alpha} = -\frac{p_i d_i'(p_i)}{d_i(p_i)}$ is the elasticity of the demand seen by the PU, and it has to be larger than 1 so that the monopoly price is finite [17]. Clearly, the α-fairness boils down to the weighted proportional fairness when $\alpha = 1$, and to selecting the SU with the highest budget when $\alpha = 0$.

2) PU's Pricing Strategy

We consider the following revenue maximization problem for the monopoly PU market:

$$\begin{aligned} \text{maximize} \quad & \sum_{i \in \mathcal{M}} (g_i + p_i \hat{s}_i) \\ \text{subject to} \quad & \hat{s}_i \le c z_i \prod_{k \ne i} (1 - z_k), \ \forall i \in \mathcal{M} \\ & \hat{s}_i = d_i(p_i), \ \forall i \in \mathcal{M} \\ & U_i(\hat{s}_i) - g_i - p_i \hat{s}_i \ge 0, \ \forall i \in \mathcal{M} \\ \text{variables} \quad & \{\mathbf{g}, \mathbf{p}, \hat{\mathbf{s}}, \mathbf{z}\}. \end{aligned} \quad (5)$$

The constraint $U_i(\hat{s}_i) - g_i - p_i \hat{s}_i \ge 0$ ensures that SUs have non-negative utility under the prices g_i and p_i, without which SUs choose not to transmit. Let g_i^*, p_i^*, $\hat{s}_i^*$ and z_i^* be the optimal solution to (5). At optimality, we have $U_i(\hat{s}_i^*) - g_i^* - p_i^* \hat{s}_i^* = 0$, $\forall i \in \mathcal{M}$; otherwise, the PU can always increase its revenue by increasing g_i^* to make the SU's net utility zero.

Lemma 2.1: The optimal prices for (5) are given by

$$g_i = U_i(\hat{s}_i) - p_i \hat{s}_i, \ \forall i \in \mathcal{M}. \quad (6)$$

Based on Lemma 2.1, (5) can be rewritten as:

$$\begin{aligned} \text{maximize} \quad & \sum_{i \in \mathcal{M}} U_i(d_i(p_i)) \\ \text{subject to} \quad & d_i(p_i) \le c z_i \prod_{k \ne i} (1 - z_k), \ \forall i \in \mathcal{M} \\ \text{variables} \quad & \{\mathbf{p}, \mathbf{z}\}. \end{aligned} \quad (7)$$

Since the utility function $U_i(\cdot)$ is increasing, the optimal solution to (7) is achieved at the point when $d_i(p_i) = c z_i \prod_{k \ne i}(1 - z_k)$, $\forall i \in \mathcal{M}$. Also, the objective function of (7) can be written as $U_i(d_i(p_i)) = \frac{1}{1-\alpha} p_i d_i(p_i)$. Since $\frac{1}{1-\alpha}$ is a constant, maximizing $\frac{1}{1-\alpha} \sum_{i \in \mathcal{M}} p_i d_i(p_i)$ is equivalent to maximizing $\sum_{i \in \mathcal{M}} p_i d_i(p_i)$, i.e., solving (5) is equivalent to solving the revenue maximization problem without considering flat prices:

$$\begin{aligned} \text{maximize} \quad & \sum_{i \in \mathcal{M}} p_i d_i(p_i) \\ \text{subject to} \quad & d_i(p_i) = c z_i \prod_{k \ne i}(1 - z_k), \ \forall i \in \mathcal{M} \\ \text{variables} \quad & \{\mathbf{p}, \mathbf{z}\}. \end{aligned} \quad (8)$$

In general, (8) is nonconvex, and therefore it is difficult to find the global optimum. Since the global optimum of (8) is
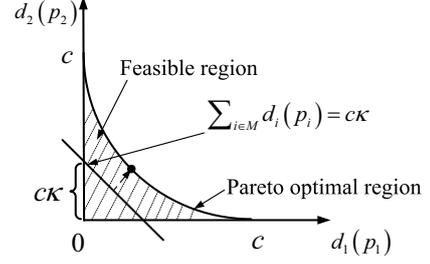

Fig. 2: A sketch of the Pareto optimal region: the case with two SUs.

Pareto optimal, we first characterize the Pareto optimal region of (8). By definition, a feasible allocation $\hat{\mathbf{s}}$ is Pareto optimal if there is no other feasible allocation $\hat{\mathbf{s}}'$ such that $\hat{s}_i' \ge \hat{s}_i$ for all $i \in \mathcal{M}$ and $\hat{s}_i' > \hat{s}_i$ for some i.

Lemma 2.2: The Pareto optimal region corresponding to (8) has the following properties:

1) The global optimum is in the Pareto optimal region.
2) The solution to (8) is Pareto optimal if and only if $\sum_{i \in \mathcal{M}} z_i = 1$.

By Lemma 2.2, for any Pareto optimal allocation $d_i(p_i)$, $\forall i \in \mathcal{M}$, we must have $\sum_{i \in \mathcal{M}} z_i = 1$. Let $\mathcal{A} = \{\mathbf{z} | \sum_{i \in \mathcal{M}} z_i = 1, z_i \ge 0, \forall i \in \mathcal{M}\}$ be the Pareto optimal region. Therefore, (8) boils down to searching for the points in $\mathcal{A}$ which can maximize (8). In light of Lemma 2.2, instead of tackling the original problem given by (8), hereafter we focus on obtaining a Pareto optimal solution to (8) that maximizes SUs' throughput subject to the SUs' budget constraints, by confining our search space to the hyperplane $\sum_{i \in \mathcal{M}} d_i(p_i) = c\kappa$, where $\kappa \in [0, 1]$ denotes the *spectrum utilization percentage* under the allocation $d_i(p_i)$, $\forall i \in \mathcal{M}$.

In what follows, we consider this "constrained" version of (8), which simplifies the original problem substantially, to finding the maximum feasible spectrum utilization κ^*, i.e., the tangent point of the hyperplane and $\mathcal{A}$, as illustrated in Figure 2:

$$\begin{aligned} \text{maximize} \quad & \sum_{i \in \mathcal{M}} p_i d_i(p_i) \\ \text{subject to} \quad & \sum_{i \in \mathcal{M}} d_i(p_i) = c\kappa \\ & d_i(p_i) = c z_i \prod_{k \ne i}(1 - z_k), \ \forall i \in \mathcal{M} \\ \text{variables} \quad & \{\mathbf{p}, \mathbf{z}, \kappa\}. \end{aligned} \quad (9)$$

We note that the solution to (9) is in general suboptimal to (8). However, by exploring the connections between the pricing strategy and the spectrum utilization, we are able to derive a closed form solution to (9) that is also a Pareto optimal solution to (8).

Proposition 2.1: For $\alpha \in (0, 1)$, the optimal solution to (9) is given by

$$\begin{aligned} p_i^* &= \left(\frac{G}{c\kappa^*}\right)^\alpha, \ \forall i \in \mathcal{M}, \\ g_i^* &= U_i(d_i(p_i^*)) - p_i^* d_i(p_i^*), \ \forall i \in \mathcal{M}, \\ \kappa^* &= \sum_{i \in \mathcal{M}} z_i^* \prod_{j \ne i}(1 - z_j^*), \\ z_i^* &= \frac{w_i}{w_i + e^{-u}}, \ \forall i \in \mathcal{M}, \end{aligned} \quad (10)$$

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[2] The PU collects the budget information of SUs in each period, which allows the PU to infer σ_i.

where $w_i = \sigma_i^{\frac{1}{\alpha}} G^{-1}$, $G = \sum_{i \in \mathcal{M}} \sigma_i^{\frac{1}{\alpha}}$, and u is the unique solution of

$$\sum_{i \in \mathcal{M}} \frac{w_i}{w_i + e^{-u}} = 1.$$

Next, we sketch a proof outline for the above proposition (The complete proof is available online [20]). First, the following result establishes the relationship between the optimal pricing strategy of (9) and κ when $\alpha \in (0, 1)$.

Lemma 2.3: Given κ, the optimal pricing strategy of (9) for $\alpha \in (0, 1)$ is given by

$$p_i^* = \left(\frac{G}{c\kappa} \right)^{\alpha}, \ \forall i \in \mathcal{M}. \qquad (11)$$

A next key step is to find κ^*. By Lemma 2.3, we have $d_i(p_i^*) = \sigma_i^{\frac{1}{\alpha}} G^{-1} c\kappa$, $\forall i \in \mathcal{M}$. Utilizing those constraints, we can find κ^* by solving the following problem:

$$\begin{aligned} \text{maximize} \quad & \kappa \\ \text{subject to} \quad & w_i \kappa \leq z_i \prod_{j \neq i} (1 - z_j), \ \forall i \in \mathcal{M} \\ \text{variables} \quad & \{\kappa, \mathbf{z}\}, \end{aligned} \qquad (12)$$

where $w_i = \sigma_i^{\frac{1}{\alpha}} G^{-1}$. Still, (12) is nonconvex, but by first taking logarithms of both the objective function and the constraints and then letting $\kappa' = \log(\kappa)$, we can transform (12) into the following convex problem:

$$\begin{aligned} \text{maximize} \quad & \kappa' \\ \text{subject to} \quad & \forall i \in \mathcal{M} \\ & \log(w_i) + \kappa' \leq \log(z_i) + \sum_{j \neq i} \log(1 - z_j), \\ \text{variables} \quad & \{\kappa', \mathbf{z}\}. \end{aligned} \qquad (13)$$

Thus, the optimal solution to (12) can be summarized by the following lemma.

Lemma 2.4: The optimal solution to (12) is given by

$$\begin{aligned} \kappa^* &= \sum_{i \in \mathcal{M}} z_i^* \prod_{j \neq i} (1 - z_j^*), \\ z_i^* &= \frac{w_i}{w_i + e^{-u}}, \ \forall i \in \mathcal{M}, \end{aligned} \qquad (14)$$

where u is the unique solution of

$$\sum_{i \in \mathcal{M}} \frac{w_i}{w_i + e^{-u}} = 1.$$

Further, when the number of SUs in the network is large, i.e., $|\mathcal{M}| \to \infty$, we can approximate κ^* by e^{-1}.

Based on Lemma 2.3 and Lemma 2.4, the optimal solution to (9) is given by Proposition 2.1, which is a Pareto optimal solution to (5). $\qquad \square$

So far we have focused on the case with $\alpha \in (0, 1)$. Now we consider the corner cases when $\alpha = 0$ and 1. Interestingly, we will see that the Pareto optimal solutions are also global optimal in those corner cases.

Corollary 2.1: When $\alpha = 0$, the Pareto optimal solution to (5) is also the global optimal solution, which is given by

$$\begin{aligned} p_i^* &= \max_{i \in \mathcal{M}} \sigma_i, \\ z_i^* &= \begin{cases} 1, \ i = k \\ 0, \ i \neq k \end{cases}, \\ g_i^* &= 0, \end{aligned} \qquad (15)$$

for all i, where $k = \arg\max_{j \in \mathcal{M}} \sigma_j$.

Corollary 2.1 implies that, when $\alpha = 0$, the PU selects the SU with the highest budget and only allows that SU to access the channel with probability 1 to maximize its revenue.

Remarks: The Pareto optimal solution in (10) converges to the globally optimal solution as α goes to zero, i.e.,

$$p_i^* = \left(\frac{G}{c\kappa^*} \right)^{\alpha} = \left(\frac{1}{c\kappa^*} \right)^{\alpha} \left(\sum_{j \in \mathcal{M}} \sigma_j^{\frac{1}{\alpha}} \right)^{\alpha} \to \max_{j \in \mathcal{M}} \sigma_j. \quad (16)$$

When $\alpha = 1$, (7) can be transformed into a convex problem by taking logarithms of the constraints, and the global optimal access probabilities of SUs in this case are

$$z_i^* = \frac{\sigma_i}{\sum_{k \in \mathcal{M}} \sigma_k}, \ \forall i \in \mathcal{M},$$

i.e., the random access probability is proportional to SU's utility level, where the revenue is dominated by the flat rate, which shares the same spirit of the results in [15]. As α goes to 1, the revenue computed by (10) also converges to the global optimal solution, since the Pareto optimal flat rates in (10) converge to the global optimal ones, which dominate the revenue.

3) Distributed Implementation of Spectrum Access Control

Based on the above study, next, we develop a distributed implementation of the pricing-based spectrum access control. Based on Proposition 2.1, the Pareto optimal solution exhibits a decentralized structure, which we now exploit to develop a distributed pricing-based spectrum access control as follows.

Algorithm 1 Distributed Pricing-based Spectrum Access Control

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1) The PU collects the budget information of SUs, i.e., $\{\sigma_i\}$.
2) The PU computes p^*, $\mathbf{g}^*$, G and u by (10), and broadcasts p^*, G and u to SUs.
3) Each SU computes z_i^* by (10) based on G and u, and infers $\mathbf{g}^*$ from its own utility and p^* by (10).
At the beginning of each period
If New SUs join the system **then**
   The new SUs inform the PU of their $\{\sigma_i\}$. Then run Steps 2 to 3.
Endif
If SUs leave the system **then**
   The leaving SUs inform the PU. Then run Steps 2 to 3.
Endif

C. Numerical Example: Pareto Optimum vs. Global Optimum

To reduce the computational complexity in solving the global optimum of (5), under slotted Aloha, we first solve (9). To examine the efficiency of this Pareto optimal solution, we exhaustively search for the global optimum of (5) to compare with the Pareto optimum, in a small network with three SUs so as to efficiently generate the true global optimum as the benchmark. Here c is 5, and each SU's σ_i is generated
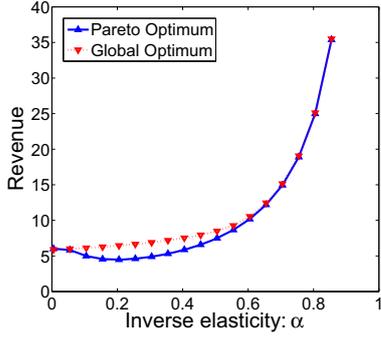
Fig. 3: Pareto optimum vs. global optimum.

uniformly in the interval $[0, 4]$ and fixed for different α for the sake of comparison. As shown in Figure 3, the Pareto optimal solution is close to the global optimal solution. Furthermore, the gap diminishes as α approaches to 1. In addition, the gap goes to zero as α goes to zero, corroborating Corollary 2.1.

D. CSMA Model for SUs' Channel Access

When the network size grows large, the spectrum utilization κ approaches $\frac{1}{e}$, and this is the typical value of "price of contention" using slotted Aloha, compared to orthogonal access that takes centralized control. It is well known that spectrum utilization can be enhanced by using CSMA. Thus motivated, next we consider a CSMA-based random access for the SUs' channel access.

When CSMA is employed, a SU can successfully access the channel after an idle period if no other SUs attempt to access the channel at the same time. Let β denote the idle time of the channel. For a given $\mathbf{z}$, the network service throughput for a large network can be approximated by [19]

$$C(T(\mathbf{z})) = \frac{T(\mathbf{z})e^{-T(\mathbf{z})}}{\beta + 1 - e^{-T(\mathbf{z})}}, \quad (17)$$

and the successful channel access probability of the ith SU can be approximated by

$$s_i(\mathbf{z}) = \frac{z_i e^{-T(\mathbf{z})}}{\beta + 1 - e^{-T(\mathbf{z})}}, \quad \forall i \in \mathcal{M}, \quad (18)$$

where $T(\mathbf{z}) = \sum_{i \in \mathcal{M}} z_i$ denotes the rate at which the SUs attempt to access the channel at the end of an idle period.

Since $C(T(\mathbf{z}))$ is maximized when $T(\mathbf{z}) = \sqrt{2\beta}$, the PU can always adjust the prices to make the SUs' channel access rates satisfy $\sum_{i \in \mathcal{M}} z_i = \sqrt{2\beta}$. In this case, $\frac{c}{1+\sqrt{2\beta}}$ can be utilized by the SUs per period, i.e., the spectrum utilization under CSMA is $\frac{1}{1+\sqrt{2\beta}}$. It follows that the revenue maximization problem under CSMA is given by:

$$
\begin{array}{ll}
\text{maximize} & \sum_{i \in \mathcal{M}} \left(g_i + p_i \hat{s}_i\right) \\
\text{subject to} & \hat{s}_i \leq \frac{c z_i e^{-\sqrt{2\beta}}}{\beta + 1 - e^{-\sqrt{2\beta}}}, \ \forall i \in \mathcal{M} \\
& \hat{s}_i = d_i(p_i), \ \forall i \in \mathcal{M} \\
& \sum_{i \in \mathcal{M}} z_i = \sqrt{2\beta}, \\
& U_i(\hat{s}_i) - g_i - p_i \hat{s}_i \geq 0, \ \forall i \in \mathcal{M} \\
\text{variables} & \{\mathbf{g}, \mathbf{p}, \hat{\mathbf{s}}, \mathbf{z}\}.
\end{array}
\quad (19)
$$

The above problem can be solved by following the same approach to (5), and the Pareto optimal solution to (19) is given by the following proposition.

Proposition 2.2: For $\alpha \in (0, 1)$, the Pareto optimal solution to (19) is given by

$$
\begin{aligned}
p_i^* &= \left(\frac{(1+\sqrt{2\beta})G}{c}\right)^{\alpha}, \ \forall i \in \mathcal{M}, \\
g_i^* &= U_i(d_i(p_i^*)) - p_i^* d_i(p_i^*), \ \forall i \in \mathcal{M}, \quad (20) \\
z_i^* &= \frac{\sigma_i^{\frac{1}{\alpha}} \sqrt{2\beta}}{G}, \ \forall i \in \mathcal{M}.
\end{aligned}
$$

Remarks: Note that (17) and (18) offer good approximations for large networks, in which case we can compare the performance of slotted Aloha and CSMA, e.g., revenue and spectrum utilization. As expected, the spectrum utilization under CSMA is higher, resulting in higher revenue from the SUs. We caution that, when the network size is small, such comparisons may not be accurate since the system capacity under CSMA is unknown. Under slotted Aloha, the results hold for an arbitrary number of SUs. Hereafter we focus on the system under slotted Aloha only.

E. The Case with Variable Spectrum Opportunity

When the spectrum opportunity c is a control parameter, there exists a tradeoff between the PU's utility and its revenue. For ease of exposition, we use the logarithmic utility function to quantify the PU's satisfaction,

$$V(c) = a \log\left(1 - \frac{c}{C}\right), \quad (21)$$

where C denotes the total length of a period, and the utility level a is a positive constant depending on the application type [12]. Given the PU's utility function, the net utility (or profit) of the PU for $\alpha \in (0, 1)$ can be expressed as:

$$R(c, \mathbf{p}) = a \log\left(1 - \frac{c}{C}\right) + \frac{1}{1-\alpha} \sum_{i \in \mathcal{M}} p_i d_i(p_i). \quad (22)$$

Based on Proposition 2.1, the Pareto optimal price is a function of the spectrum opportunity c. Then, the optimal c^* can be found by solving the following problem:

$$
\begin{array}{ll}
\text{maximize} & a \log\left(1 - \frac{c}{C}\right) + b c^{1-\alpha} \\
\text{subject to} & 0 \leq c \leq C \\
\text{variable} & \{c\},
\end{array}
\quad (23)
$$

where $b = \frac{1}{1-\alpha} \kappa^{*1-\alpha} G^{\alpha}$ is a positive constant.

It is easy to see that the optimal c^* is determined by the first order condition and the boundary conditions, and is the solution to

$$-\frac{a}{C - c^*} + \frac{b}{1-\alpha} c^{*-\alpha} = 0. \quad (24)$$

To get a more concrete sense, we plot two possible curves of (22) for different b in Figure 4. In this example, we set $a = 20$, $C = 100$, $\alpha = 0.5$ and $|\mathcal{M}| = 20$. Each SU's σ_i is generated uniformly in the interval $[\sigma_{\min}, \sigma_{\max}]$. For the two realizations of σ_i, $\forall i \in \mathcal{M}$, the optimal tradeoff decision, corresponding to the highest point of each curve within $(0, C)$, increases with b. Intuitively speaking, the PU would allocate more spectrum opportunity to those SUs who would pay more.
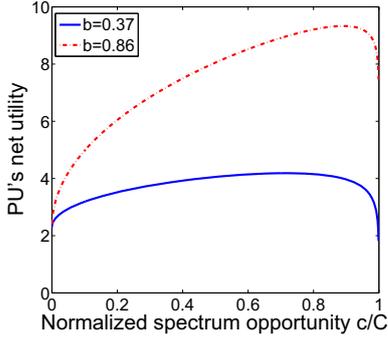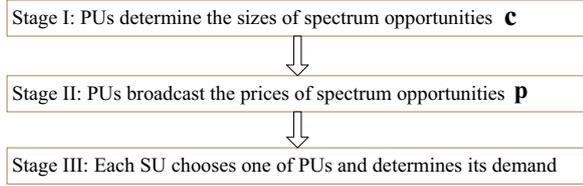
Fig. 4: PU's profit under different b.

Fig. 5: A three-stage Stackelberg game.

III. MULTIPLE PU MARKET

A. System Model

When there are multiple PUs in a cognitive radio network, they compete with each other in terms of prices and spectrum opportunities, in order to maximize their net utilities. On the PUs' side, flat prices are functions of usage prices based on Lemma 2.1. On the SUs' side, since PUs set flat prices based on Lemma 2.1, each SU wishes to choose the PU with the lowest usage price to improve its transmission rate. Thus, we focus on usage prices in what follows, and the corresponding flat prices can be obtained accordingly.

We cast the competition among PUs as a three-stage Stackelberg game, as summarized in Figure 5, where the PUs and SUs adapt their decisions dynamically to reach an equilibrium point. The PUs first simultaneously determine their available spectrum opportunities in Stage I, and then simultaneously announce the prices to SUs in Stage II. Finally, each SU accesses only one PU's channel to maximize its throughput in Stage III. Here, we consider a set $\mathcal{N}$ of homogeneous PUs (i.e., PUs with the same type of utility function defined in (21)). We assume that all SUs are within the intersection of those PUs' coverage areas shown in Figure 1(b).

In the sequel, we focus on the game for $\alpha \in (0, 1)$, and use the index $i \in \mathcal{M}$ for SUs and the index $j \in \mathcal{N}$ for PUs.

B. Backward Induction for the Three-stage Game

We use the backward induction method [17] to analyze the game. First, we start with Stage III and analyze SUs' behaviors given PUs' spectrum opportunities and prices. Then we will turn our focus to Stage II and analyze how PUs determine prices given spectrum opportunities and the possible reactions of SUs in Stage III. Finally, we will study how PUs determine spectrum opportunities given the possible reactions in Stage II and III.

1) Channel Selection in Stage III

In this stage, each SU determines which PU's channel to access based on the set of prices $\mathbf{p}$. The admission of SUs also depends on the available spectrum opportunities $\mathbf{c}$ in Stage I. Since (4) decreases with price p_j, the ith SU would choose the jth PU's channel if $p_j = \min_{k \in \mathcal{N}} p_k$.

Given the set of prices $\mathbf{p}$, the total demand of SUs in the jth PU's channel can be written as

$$D_j(p_j, \mathbf{p_{-j}}) = \sum_{i \in \mathcal{M}_j} d_i(p_j) = \sum_{i \in \mathcal{M}_j} \sigma_i^{\frac{1}{\alpha}} p_j^{-\frac{1}{\alpha}}, \quad (25)$$

where $\mathcal{M}_j$ denotes the set of SUs choosing the jth PU, and $\mathbf{p_{-j}}$ denotes the set of prices of PUs other than the jth PU. Both $\mathcal{M}_j$ and D_j depend on prices $\mathbf{p}$, and are independent of $\mathbf{c}$. Therefore, the demand function can be written as

$$D_j(p_j, \mathbf{p_{-j}}) = \begin{cases} \frac{G}{|\mathcal{J}|} p_j^{-\frac{1}{\alpha}}, & j \in \mathcal{J} \\ 0, & j \notin \mathcal{J} \end{cases}, \quad (26)$$

where G is defined in Proposition 2.1, and $\mathcal{J} = \{j | p_j = \min_{k \in \mathcal{N}} p_k, j \in \mathcal{N}\}$ denotes the set of PUs with the smallest price in $\mathcal{N}$. In this paper, we assume that SUs randomly pick one PU in $\mathcal{J}$ with equal probability.

Given the size of available spectrum opportunities c_j, the jth PU always adjusts its price to make the demand of SUs equal to the supply so as to maximize its revenue (based on Proposition 2.1). It follows that at the equilibrium point,

$$D_j(p_j, \mathbf{p_{-j}}) = c_j \kappa_j^*, \quad \forall j \in \mathcal{N}, \quad (27)$$

where κ_j^* denotes the maximum feasible spectrum utilization of the jth PU's channel, and is given by Lemma 2.4. Since κ_j^* depends on the budgets of SUs, it is difficult for the PU to decide whether or not to admit new SUs based on the current demand. Since κ_j^* approaches e^{-1} when the number of SUs is reasonably large [19], we will approximate using this asymptote. Accordingly, (27) can be rewritten as

$$D_j(p_j, \mathbf{p_{-j}}) = \frac{c_j}{e}, \quad \forall j \in \mathcal{N}. \quad (28)$$

2) Pricing Competition in Stage II

In this stage, the PUs determine their pricing strategies while considering the demands of SUs in Stage III, given the available spectrum opportunities $\mathbf{c}$ in Stage I. The profit of the jth PU can be expressed as

$$R_j(c_j, p_j, \mathbf{c_{-j}}, \mathbf{p_{-j}}) = a \log\left(1 - \frac{c_j}{C}\right) + \frac{1}{1-\alpha} p_j D_j(p_j, \mathbf{p_{-j}}). \quad (29)$$

Since c_j is fixed at this stage, the jth PU is only interested in maximizing the revenue $p_j D_j(p_j, \mathbf{p_{-j}})$. Obviously, if the jth PU has no available spectrum to sell, i.e., $c_j = 0$, it would not compete with other PUs by price reduction to attract SUs. For convenience, define $\mathcal{N}' = \{j | c_j > 0, \forall j \in \mathcal{N}\}$ as the set of PUs with positive spectrum opportunity.

Game at Stage II: The competition among PUs in this stage can be modeled as the following game:

- *Players*: PUs in the set $\mathcal{N}'$;
- *Strategy*: each PU can choose a price p_j from the feasible set $\mathcal{P} = [p_{min}, \infty)$;
- *Objective function*: $p_j D_j(p_j, \mathbf{p_{-j}}), j \in \mathcal{N}'$,

where p_{min} denotes the minimum price that each PU can choose, and is determined by (10) at $c = C$.

Lemma 3.1: A necessary condition for PUs to achieve an equilibrium price is $\sum_{j \in \mathcal{N}'} c_j \leq C$.

Proposition 3.1: There exists a unique equilibrium price. Moreover, the equilibrium price is given by $p_j^* = p^*$, $\forall j \in \mathcal{N}'$, where

$$p^* = \left(\frac{eG}{\sum_{j \in \mathcal{N}'} c_j}\right)^\alpha, \tag{30}$$

and $\sum_{j \in \mathcal{N}'} c_j \leq C$.

Proposition 3.1 shows that no PU would announce a price higher than its competitors to avoid losing most or all of its demand to its competitors, and the optimal strategy is to make the same decision as its competitors. Since $c_j = 0, \forall j \notin \mathcal{N}'$, the equilibrium price (30) can be written as:

$$p^* = \left(\frac{eG}{\sum_{j \in \mathcal{N}} c_j}\right)^\alpha, \tag{31}$$

where $\sum_{j \in \mathcal{N}} c_j \leq C$.

3) Spectrum Opportunity Allocation in Stage I

In this stage, PUs need to decide the optimal spectrum opportunities to maximize their profits. Based on Proposition 3.1, the jth PU's profit can be written as

$$
\begin{aligned}
R_j(c_j, \mathbf{c_{-j}}) &= a \log\left(1 - \frac{c_j}{C}\right) + \frac{c_j}{e(1-\alpha)}\left(\frac{eG}{\sum_{k \in \mathcal{N}} c_k}\right)^\alpha \\
&= \frac{e^{\alpha-1}G^\alpha}{1-\alpha}\left(\hat{a}\log\left(1 - \frac{c_j}{C}\right) + \frac{c_j}{(\sum_{k \in \mathcal{N}} c_k)^\alpha}\right),
\end{aligned}
$$

where $\hat{a} = (1-\alpha)e^{1-\alpha}G^{-\alpha}a$. For convenience, define $\hat{R}_j(c_j, \mathbf{c_{-j}}) = \hat{a}\log\left(1 - \frac{c_j}{C}\right) + \frac{c_j}{(\sum_{k \in \mathcal{N}} c_k)^\alpha}$. Since $\frac{e^{\alpha-1}G^\alpha}{1-\alpha}$ is a constant, maximizing $R_j(c_j, \mathbf{c_{-j}})$ is equivalent to maximizing $\hat{R}_j(c_j, \mathbf{c_{-j}})$.

Game at Stage I: The competition among PUs in this stage can be modeled as the following game:

- *Players*: PUs in the set $\mathcal{N}$;
- *Strategy*: PUs will choose $\mathbf{c}$ from the feasible set $\mathcal{C} = \{\mathbf{c} | \sum_{j \in \mathcal{N}} c_j \leq C, c_j \in [0, C), \forall j \in \mathcal{N}\}$;
- *Objective function*: $\hat{R}_j(c_j, \mathbf{c_{-j}}), \forall j \in \mathcal{N}$.

To find the spectrum opportunity equilibrium of the game at this stage, we first examine the strategy of the jth PU given other PUs' decisions. Due to the concavity of $\hat{R}_j(c_j, \mathbf{c_{-j}})$ in c_j, the existence of equilibrium can be readily shown based on [18]. Also, we can obtain the best response strategy of the jth PU by checking the first order condition $\partial \hat{R}_j(c_j, \mathbf{c_{-j}})/\partial c_j = 0$ and the boundary conditions (see our technical report [20] for a detailed derivation considering several cases associated with decision making threshold). As expected, the best response strategy for the jth PU depends on $\hat{a}$ and its competitors' decision $\mathbf{1^T c_{-j}}$. Let c^L and c^H be the thresholds for PUs'

decision making associated with $0 < \hat{a} \leq C^{1-\alpha}$ and $\hat{a} > C^{1-\alpha}$ respectively. They are given explicitly by $c^L = \frac{1}{2\alpha}(\alpha - 1 + \sqrt{(\alpha-1)^2 + 4\alpha\hat{a}C^{\alpha-1}})$ and $c^H = \left(\frac{\hat{a}}{C}\right)^{-\frac{1}{\alpha}}$ (derivation in [20]). We now establish the response strategy for the PUs.

Proposition 3.2: The best response strategy for the jth PU in the above game is outlined as follows:

1) **The case with $0 < \hat{a} \leq C^{1-\alpha}$:**
   If $\mathbf{1^T c_{-j}} \in [0, Cc^L]$, then c_j^* is the solution to

   $$(c_j^* + \mathbf{1^T c_{-j}})^{-\alpha} - \alpha c_j^*(c_j^* + \mathbf{1^T c_{-j}})^{-\alpha-1} - \frac{\hat{a}}{C - c_j^*} = 0; \tag{32}$$

   If $\mathbf{1^T c_{-j}} \in [Cc^L, C]$, then $c_j^* = C - \mathbf{1^T c_{-j}}$.

2) **The case with $\hat{a} > C^{1-\alpha}$:**
   If $\mathbf{1^T c_{-j}} \in [0, c^H]$, then c_j^* is the solution to

   $$(c_j^* + \mathbf{1^T c_{-j}})^{-\alpha} - \alpha c_j^*(c_j^* + \mathbf{1^T c_{-j}})^{-\alpha-1} - \frac{\hat{a}}{C - c_j^*} = 0; \tag{33}$$

   If $\mathbf{1^T c_{-j}} \in [c^H, C]$, then $c_j^* = 0$.

As expected, the equilibrium of Game at Stage I depends on $\hat{a}$, α, c^L, and the number of PUs. For convenience, define $\mathcal{N}_1 = \{j | c_j^*$ is the solution to (32), $\forall j \in \mathcal{N}\}$ and $\mathcal{N}_2 = \{j | c_j^* = C - \mathbf{1^T c_{-j}}, \forall j \in \mathcal{N}\}$. Observe that $\mathcal{N}_1$ represents the set of PUs whose competitors' decision is below the threshold Cc^L, while $\mathcal{N}_2$ represents the set of PUs whose competitors' decision is above the threshold Cc^L. The spectrum opportunity equilibria of Game at Stage I are given by the following proposition.

Proposition 3.3: The spectrum opportunity equilibria in Stage I are summarized as follows:

1) **The case with $0 < \hat{a} \leq C^{1-\alpha}$:**
   a) If $c^L \leq \frac{|\mathcal{N}|-1}{|\mathcal{N}|}$ (i.e., $\mathcal{N}_1 \neq \emptyset$ and $\mathcal{N}_2 \neq \emptyset$), there exist infinitely many spectrum opportunity equilibria satisfying

   $$\sum_{j_2 \in \mathcal{N}_2} c_{j_2}^* = C - |\mathcal{N}_1|C(1 - c^L),$$

   $$c_{j_1}^* = C(1 - c^L), \forall j_1 \in \mathcal{N}_1,$$

   $$C - c_{j_2}^* \geq Cc^L, \forall j_2 \in \mathcal{N}_2.$$

   b) If $c^L > \frac{|\mathcal{N}|-1}{|\mathcal{N}|}$ (i.e., $\mathcal{N}_1 = \mathcal{N}$ and $\mathcal{N}_2 = \emptyset$), there exists a unique spectrum opportunity equilibrium such that $c_j^* = c^*, \forall j \in \mathcal{N}$, where c^* is the solution to

   $$(|\mathcal{N}|c^*)^{-\alpha}(1 - \frac{\alpha}{|\mathcal{N}|}) = \frac{\hat{a}}{C - c^*}. \tag{34}$$

2) **The case with $\hat{a} > C^{1-\alpha}$:**
   There exists a unique spectrum opportunity equilibrium such that $c_j^* = c^*, \forall j \in \mathcal{N}$, and lies in the strict interior $(0, \frac{1}{|\mathcal{N}|}\left(\frac{\hat{a}}{C}\right)^{-\frac{1}{\alpha}})$, where c^* is the solution to

   $$(|\mathcal{N}|c^*)^{-\alpha}(1 - \frac{\alpha}{|\mathcal{N}|}) = \frac{\hat{a}}{C - c^*}. \tag{35}$$

Based on Proposition 3.3, we can see that there exists a *threshold* for the number of PUs, denoted by N_{PU}, in the case with $0 < \hat{a} \leq C^{1-\alpha}$, where N_{PU} is given by $\frac{1}{1-c^L}$.

Accordingly, Proposition 3.3 can be treated as a criterion for PUs to decide whether to join the competition or not, because each PU can calculate the pricing equilibrium when it gathers the necessary information based on Proposition 3.3. In the case with $0 < \hat{a} \le C^{1-\alpha}$, it needs to check whether the condition $N_{PU} > |\mathcal{N}|$ holds. This is because if $N_{PU} \le |\mathcal{N}|$ after it joins the competition, the pricing equilibrium will be $p_{\min}$, which may make it unprofitable to offer its spectrum to SUs.

C. An Algorithm for Computing Equilibria

To achieve the equilibrium of the dynamic game, we present an iterative algorithm for each PU. Based on Lemma 3.1, if $\sum_{j\in\mathcal{N}} c_j > C$, the spectrum allocation is inefficient, i.e., there always exists some PU whose supply is larger than the demand. Thus, each PU first updates its spectrum opportunity based on the demand to fully utilize its spectrum. Once the necessary condition $\sum_{j\in\mathcal{N}} c_j \le C$ is satisfied, each PU can update its spectrum opportunity in the Stage I by Proposition 3.2. We assume that the "total budget" G of SUs is available to each PU. The proposed algorithm for computing the market equilibrium is summarized in Algorithm 2. Based on Lemma 3.1, Proposition 3.2 and Proposition 3.3, Algorithm 2 converges to the equilibrium of the game, as also verified by simulation.

Algorithm 2 Computing the equilibrium of the multiple PU market

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Initialization: Each PU collects the budget information of SUs, i.e., $\{\sigma_i\}$.
At the beginning of each period
1) **If** $\sum_{j\in\mathcal{N}} c_j > C$ **then**
    Each PU sets $c_j = D_j(p_j, \mathbf{p_{-j}})$, and broadcasts c_j.
  Else
    Each PU sets c_j based on Proposition 3.2, and broadcasts c_j.
  Endif
2) Each PU sets $p_j = \max\left(p_{\min}, \left(\frac{eG}{\sum_{j\in\mathcal{N}} c_j}\right)^{\alpha}\right)$, and broadcasts p_j.
3) Each SU randomly chooses a PU's channel with the lowest price.
4) Each PU admits new SUs when $c_j > D_j(p_j, \mathbf{p_{-j}})$.
5) Steps 1 to 4 are repeated in each period.

Remarks: Algorithm 2 is applicable to the scenarios, where the PUs can vary their spectrum opportunities. When the spectrum opportunities are fixed, the three-stage game reduces to a two-stage game without the stage of spectrum opportunity allocation. In this case, the equilibrium of the game is given by Proposition 3.1.

D. Numerical Examples: Equilibria of Competitive PUs

Based on Lemma 3.1, there always exists an equilibrium in the feasible region (i.e., $\sum_{j\in\mathcal{N}} c_j \le C$). In the following, we first illustrate the existence and uniqueness of the equilibrium by (32) for two PUs, in the case with $0 < \hat{a} \le C^{1-\alpha}$ and $c^L > 0.5$. Then we consider a more general system model
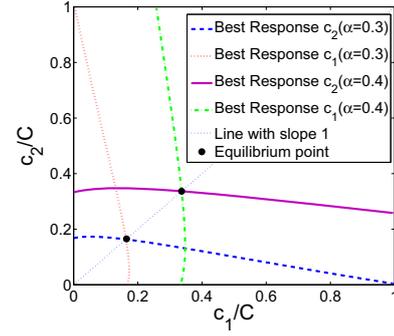

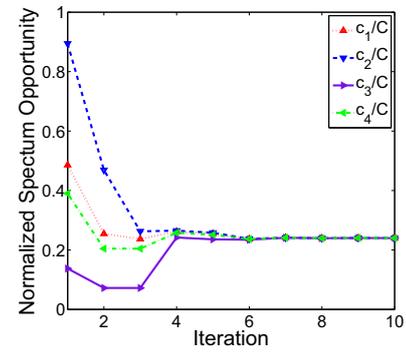Fig. 6: Existence/uniqueness of spectrum opportunity equilibrium.

Fig. 7: Convergence of spectrum opportunity equilibrium.

with four PUs and examine the convergence of spectrum opportunities. Finally, we demonstrate how the equilibrium price evolves under different elasticities of SUs and different numbers of PUs. In each experiment, C is equal to 20, and each SU's budget σ_i is generated uniformly in the interval $[0, 4]$, and is fixed for different α for the sake of comparison.

The existence and uniqueness of the equilibrium, corresponding to the competitive spectrum opportunity of two PUs, is illustrated in Figure 6. In particular, we change the inverse elasticity (i.e., α) of SUs from 0.3 to 0.4 in order to show how the spectrum opportunity equilibrium evolves, when $0 < \hat{a} \le C^{1-\alpha}$ and $c^L > 0.5$. Based on Proposition 3.3, there exists a unique spectrum opportunity equilibrium, which is shown in Figure 6. Furthermore, we observe that the spectrum equilibrium increases with α, and lies on the line with slope one, which is due to the symmetry of the best response functions.

In Figure 7, we examine the convergence of spectrum opportunities in the case of four PUs and 200 SUs by using Algorithm 2. Here, we choose $\alpha = 0.3$ and $a = 30$ such that $0 < \hat{a} \le C^{1-\alpha}$ and $c^L > 0.75$. As illustrated in Figure 7, the sum of initial normalized spectrum opportunities is greater than 1, in which case there is no equilibrium point based on Lemma 3.1. Each PU then updates its offered spectrum opportunity based on its current demand (this process corresponds to the iterations from 1 to 3 in Figure 7). Once the total supply is within the feasible region, each PU adjusts
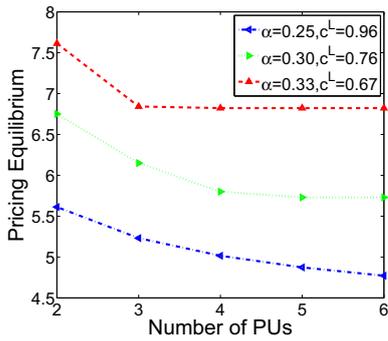
Fig. 8: Price equilibrium corresponding to different elasticities of SUs and different numbers of PUs.

its supply based on Proposition 3.2. Based on Proposition 3.3, there exists a unique equilibrium, which is verified in Figure 7.

Figure 8 depicts how the equilibrium price evolves under different elasticities of SUs and different numbers of PUs. Specifically, we choose $a = 30$ and $|\mathcal{M}| = 200$. For each α, the equilibrium price decreases with the number of PUs, due to more competition among PUs. In other words, SUs can benefit from the competition among PUs. Moreover, it can be seen that the equilibrium price approaches $p_{\min}$ as the number of PUs increases.

IV. CONCLUSION

This paper studied pricing-based spectrum access control in cognitive radio networks, where SUs contend for channel usage by random access when the PU's channel is available. We studied two models: one with a monopoly PU market and the other with a multiple PU market. For the monopoly PU market model, we used the revenue maximization approach to characterize the appropriate choice of flat and usage prices, and developed a Pareto optimal solution, which was shown to be near-optimal. More importantly, this Pareto optimal solution exhibits a decentralized structure, i.e., the Pareto optimal pricing strategy and access probabilities can be computed by the PU and the SUs locally. We also analyzed a PU profit maximization problem by examining the tradeoff between the PU's utility and its revenue.

We then studied the competition of PUs in the multiple PU market by modeling the competition as a three-stage Stackelberg game among PUs, in terms of access prices and the offered spectrum opportunities. We showed that the number of equilibria has a threshold structure: if the number of PUs is greater than the threshold, there exist infinitely many equilibria, where the equilibrium price reduces to the minimum price that may cut down PUs' profit; otherwise, there exists a unique equilibrium. We further provided an iterative algorithm to find the equilibrium.

In this paper, we have studied thoroughly pricing-based distributed spectrum access control for SUs, for the market model with homogeneous PUs. A natural next step is to explore the model with heterogeneous PUs. Another interesting direction

is to investigate transient behaviors corresponding to dynamic spectrum access, in the presence of spectrum hole dynamics.

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