Revenue Sharing among ISPs in Two-Sided Markets

Yuan Wu[1], Hongseok Kim[2], Prashanth H. Hande[3], Mung Chiang[4], Danny H.K. Tsang[1]
[1]Department of Electronic and Computer Engineering, Hong Kong University of Science and Technology
[2]Bell Laboratories, Alcatel-Lucent, Murray Hill, NJ
[3]Qualcomm Inc. NJ
[4]Department of Electrical Engineering, Princeton University

***Abstract*—In this paper, we study the revenue sharing and rate allocation for Internet Service Providers (ISPs) that jointly provide network connectivity between content providers and end-users. Without colluding, each ISP may selfishly set a high transit-price to cover its cost and maximize its own profit, which inevitably results in a loss in social profit. We model this noncooperative interaction between an "eyeball" ISP and a "content" ISP as a Stackelberg game and quantify the resulting loss in social profit. To recover the profit loss, we propose a revenue sharing contract between ISPs by modeling them as a supply chain to deliver traffic in a two-sided market. Parameterized by the profit division factor, the sharing contract coordinates ISPs' objectives such that they aim to maximize the social profit self-incentively. We further propose a Nash bargaining process to determine the profit division factor such that all ISPs are simultaneously better off compared to the noncooperative equilibrium.**

I. Introduction

Two-sided market, which charges both Content Providers (CPs) and End Users (EUs) for access to Internet, is emerging as a useful approach for ISPs to increase their profits and for CPs to increase demand level of EUs. Profit maximization in a two-sided market, where EUs and CPs are charged by a single ISP for interconnection, was recently studied in [1]. The assumption of a single representative ISP providing connectivity does not match the reality where multiple ISPs cooperate to provide end-to-end connectivity service between CPs and EUs. In addition to technology issues, interconnection among ISPs also depends on inter-charging/profit sharing arrangements because ISPs with privately-owned infrastructures usually care about their profits and have no incentives to benefit others. Meanwhile, most commercial traffic is asymmetric [2] [3], e.g., originating from CPs and terminating at EUs, which implies the biased positions of the interconnected ISPs on a traffic delivery chain. For example, an "eyeball" ISP serving a large population of EUs can be more powerful in requiring a transit-price to terminate traffic in its network. Some powerful CPs initiate the construction of their own networks and become ISPs for themselves. In this situation, these ISPs are more powerful in determining the transit-price paid to relatively weak ISPs. A brief motivating example is as follows (with details in [10]). Two interconnected ISPs with their own cost structures provide connectivity from CPs to EUs. To compensate for its cost, each (selfish) ISP must require a transit-price no smaller than its marginal cost for traffic delivery, which may adversely influence the corresponding traffic demand from another ISP. However, if one ISP lowers down its transit-price to encourage the other ISP's traffic demand, their social profit will increase. What mechanism can motivate these selfish ISPs to cooperate? And how do these ISPs reach a profit sharing agreement according to their costs and strategic bargaining? We answer these two questions in Theorem 1 and Theorem 5, respectively. Meanwhile, in the previous example, both ISPs obtain profits by charging customers in a two-sided market, i.e., one ISP directly charges CPs and the other ISP directly charges EUs. The CPs and EUs have different price elasticities and utility levels for data. Therefore, how these parameters influence the ISPs' inter-pricing strategies and equilibrium profits? We consider the case where the EU-facing ISP has dominant power. The characterization of the profit loss is obtained in Theorems 2, 3 and 4.

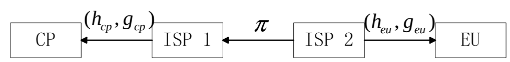

Fig. 1. ISP 1 (ISP 2) charges CP (EU) with usage-price h_{cp} (h_{eu}) and flat-price g_{cp} (g_{eu}). ISP 2 charges ISP 1 with transit-price π for traffic delivery.

To address the above questions, we consider a typical network where two ISPs provide access services to CP and EU, respectively, as shown in Figure 1. The EU-facing ISP (ISP 2) charges the CP-facing ISP (ISP 1) with transit-price for traffic delivery. We model the service rate provisioning of CP and traffic demand of EU in a similar way as those in [1], i.e., both of CP and EU's utilities are measured in terms of the effective data rate, thus rendering a two-sided market.

Assume collusion between these two ISPs. We first model the bilateral interaction between them as a supply chain and apply the revenue sharing contract [4] to this chain. The main idea of the proposed contract is that the dominant ISP requires a transit-price lower than its marginal cost for traffic delivery. Meanwhile, to compensate for its profit loss, the dominant ISP claims a lump-sum sharing of the other ISP's income. By appropriately structuring the transit-price, this simple contract efficiently encourages coordination between the ISPs such that the social profit is maximized.

As the other extreme case of ISPs' interaction, we analyze the noncooperative model in which ISPs fail to collude and each ISP selfishly maximizes its own profit. In particular, Stackelberg game fits well as a model here because the dominant ISP on the traffic chain can naturally be considered as the game leader who determines the transit price prior to the decision of the follower ISP. We quantify the social profit loss due to this noncooperation.

Building upon the noncooperative model, we design a profit division scheme to divide the optimal social profit obtained from the revenue sharing contract. We first quantify the range for the viable profit division factor such that each ISP can be better off compared to the noncooperative equilibrium. We then use the asymmetric Nash Bargaining Solution (NBS) [5] to determine the optimal division factor.

A. Related Work

Multiple ISPs pricing in *one-sided* market has been extensively studied. For example, [3] studied the equilibrium of pricing competition among ISPs and obtained the social optimality with the repeated game model. However, it is noticed that the social optimality alone cannot guarantee that each individual ISP will be better off compared to the result when they are noncooperative. [6] focused on the peering strategy used by local ISPs. However, it may be of no economic interest for an ISP with large consumer-size to establish peering relationship with another ISP with a small consumer-size. [7] further considered the impact of application layer routing on ISPs' peering strategy. [8] used Shapley value to model selfish ISPs' routing and interconnecting decisions. [9] proposed multiple ISPs to share revenue proportional to their local costs. However, [8] and [9] did not consider the issues of the social optimality and the relevant loss in social profit when ISPs are noncooperative (both of which are our focuses in this work). On the other hand, [1] studied the optimal pricing structure and rate allocation for a single representative monopolistic ISP in a *two-sided* market, but [1] did not analyze the interaction between ISPs.

II. Network Scenario and Two-Sided Market

We consider a scenario of two ISPs as shown in Figure 1. Traffic originates from CP (whose access connectivity is provided by ISP 1) and terminates at EU (whose access service is provided by ISP 2). Both ISP 1 and ISP 2 aim to maximize their own profits. ISP 1 uses the combination of usage-price h_{cp} and flat-price g_{cp} to charge CP. Meanwhile, ISP 2 uses the combination of usage-price h_{eu} and flat-price g_{eu} to charge EU. ISPs 1 and 2 have their own marginal costs c_1 and c_2 for traffic delivery, respectively. In particular, we will focus on the case where ISP 2 is dominant and can determine the transit-price π charged to ISP 1 prior to ISP 1's decision. For example, ISP 2 provides connectivity to a large population-size of EUs and is thus more powerful in determining the transit-price than ISP 1. Therefore, ISP 2 (ISP 1) is the leader (follower) in the following noncooperative game analyzed in Section IV. Our model can also capture the reverse scenario where ISP 1 is dominant and determines the transit-price π paid to ISP 2 prior to ISP 2's decision, e.g., ISP 1 provides access to a very popular CP.

We model CP's service rate provisioning and EU's traffic rate demand (in terms of the effective data rate) as follows. Given ISP 1's prices $\{h_{cp}, g_{cp}\}$, CP decides its service rate by solving the following problem

$$\textbf{(CP-P):}\ \max_{y\geq 0}\ \sigma_{cp}u_{cp}(y) - h_{cp}y - g_{cp}, \tag{1}$$

where y denotes CP's service rate provisioning and σ_{cp} denotes CP's utility level (e.g., σ_{cp} indicates the popularity of the content). A common example of utility function is $u_{cp}(y) = \frac{1}{1-\alpha_{cp}}y^{1-\alpha_{cp}}, 0 < \alpha_{cp} < 1$. In this case, CP's service rate provisioning function becomes $D_{cp}(h_{cp}) = (\frac{\sigma_{cp}}{h_{cp}})^{\frac{1}{\alpha_{cp}}}$. CP's price elasticity can be expressed as $\epsilon_{cp} = \frac{dD_{cp}}{dh_{cp}}\frac{h_{cp}}{D_{cp}} = \frac{1}{\alpha_{cp}}$, which is a normalized metric characterizing how much the rate provisioning changes as the price changes. A low price elasticity implies it is more difficult for ISP 1 to change CP's service rate provisioning by changing its usage-price h_{cp}.

Similarly, given ISP 2's prices $\{h_{eu}, g_{eu}\}$, EU decides its traffic rate demand by solving the following problem

$$\textbf{(EU-P):}\ \max_{y\geq 0}\ \sigma_{eu}u_{eu}(y) - h_{eu}y - g_{eu}, \tag{2}$$

where σ_{eu} denotes EU's utility level (e.g., σ_{eu} represents EU's desire to obtain the data). EU's traffic rate demand can thus be expressed as $D_{eu}(h_{eu}) = (\frac{\sigma_{eu}}{h_{eu}})^{\frac{1}{\alpha_{eu}}}$ for the utility function $u_{eu}(y) = \frac{1}{1-\alpha_{eu}}y^{1-\alpha_{eu}}, 0 < \alpha_{eu} < 1$. EU's price elasticity can be expressed as $\epsilon_{eu} = \frac{dD_{eu}(h_{eu})}{dh_{eu}}\frac{h_{eu}}{D_{eu}(h_{eu})} = \frac{1}{\alpha_{eu}}$.

Let x denote the effective data rate which can be successfully delivered from CP to EU. Obviously, $x \leq \min\{D_{cp}(h_{cp}), D_{eu}(h_{eu})\}$ holds.

III. Collusive ISPs with Revenue Sharing

As described in Section II, ISP 1 and ISP 2 have their own interests. Specifically, ISP 1 aims to maximize its profit as $(h_{cp}x + g_{cp}) - c_1x - \pi x$, and ISP 2 aims to maximize its profit as $(h_{eu}x + g_{eu}) - c_2x + \pi x$, where $x \leq \min\{D_{cp}(h_{cp}), D_{eu}(h_{eu})\}$. Notice that two ISPs' profits are conflicting, thus ISPs 1 and 2 usually do not cooperate.

The revenue sharing contract provides an efficient mechanism to encourage coordination among different agents [4]. We will soon see how the sharing contract makes both ISPs better off compared to the case where they selfishly maximize their own profits (in section V). In this section, we suppose that ISP 1 and ISP 2 reach a sharing contract which is specified by $\{\theta, \gamma, \pi\}$. θ denotes the portion of income that ISP 1 retains for itself (and thus shares $1-\theta$ of its income with ISP 2). Similarly, γ denotes the portion of income that ISP 2 retains for itself (and thus shares $1-\gamma$ of its income with ISP 1). As described before, π denotes the transit-price ISP 2 charges ISP 1 for delivering traffic from CP to EU. Under this contract, ISP 1's profit can be expressed as

$$R_1 = (h_{cp}x + g_{cp})\theta + (h_{eu}x + g_{eu})(1-\gamma) - c_1x - \pi x. \tag{3}$$

Meanwhile, ISP 2's profit can be expressed as

$$R_2 = (h_{cp}x + g_{cp})(1-\theta) + (h_{eu}x + g_{eu})\gamma - c_2x + \pi x. \tag{4}$$

The social profit R_s of the two ISPs (i.e., $R_s = R_1 + R_2$) is

$$R_s = (h_{cp}x + g_{cp}) + (h_{eu}x + g_{eu}) - c_1x - c_2x. \tag{5}$$

Theorem *1:* Given the parameterized sharing contract $\{\theta, \gamma, \pi\}$, if the sharing factor is given by $\theta = 1 - \gamma$, and the transit-price is given by $\pi = \theta(c_1 + c_2) - c_1$, then ISPs 1 and 2 are coordinated in the sense that maximizing the social profit also maximizes the profit of each ISP. Specifically, the profit of ISP 1 can be expressed as $R_1 = \theta R_s$, and the profit of ISP 2 can be expressed as $R_2 = (1 - \theta)R_s$. Please refer to [10] for the detailed proof (and also the proofs for the rest theorems in this work).

The intuition behind the conditions in Theorem 1 is as follows. First, the transit-price $\pi = \theta(c_1 + c_2) - c_1 \leq c_2$ because of $0 \leq \theta \leq 1$. It implies that ISP 2 charges ISP 1 with the transit-price smaller than its marginal cost. To compensate for its loss, however, ISP 2 requires a portion of ISP 1's income. Second, the proposed sharing contract indicates that if ISP $i, i = 1, 2$, wants to claim a large portion of the social income (i.e., $(h_{cp}x + g_{cp}) + (h_{eu}x + g_{eu})$), then it has to afford a large portion of the entire cost (i.e., $c_1 + c_2$) accordingly.

A. Pricing Strategy for Optimal Social Profit

With the proposed sharing contract, we first consider the Social Profit Maximization (SPM) problem as follows.

$$\textbf{(SPM):}\ \max\ (h_{cp}x + g_{cp}) + (h_{eu}x + g_{eu}) - c_1 x - c_2 x$$

$$\text{subject to: } \sigma_{cp}u_{cp}(x) \geq h_{cp}x + g_{cp}, \quad (6)$$

$$\sigma_{eu}u_{eu}(x) \geq h_{eu}x + g_{eu}, \quad (7)$$

$$x \leq \min\{D_{cp}(h_{cp}), D_{eu}(h_{eu})\}, \quad (8)$$

$$x \geq 0, h_{cp} \geq 0, h_{eu} \geq 0, g_{cp} \geq 0, g_{eu} \geq 0. \quad (9)$$

Constraint (6) and constraint (7) guarantee nonnegative net-utilities for CP and EU, respectively. Constraint (8) restricts the effective traffic rate based on both CP's service rate provisioning and EU's rate demand. Problem (**SPM**) is a nonconvex problem, which is difficult to solve in general. However, by noticing that constraints (6) and (7) should be binding at the optimum, we can first consider an Equivalent problem for problem (**SPM**) as follows (named as **(SPM-E)**).

$$\max_{x \geq 0}\ R_s(x) = \max_{x \geq 0}\ \sigma_{cp}u_{cp}(x) + \sigma_{eu}u_{eu}(x) - c_1 x - c_2 x \quad (10)$$

Problem **(SPM-E)** is a convex problem, and thus the first order optimality condition is applicable. Let x^* denote the optimal rate allocation problem **(SPM-E)**, we have condition **(C1)**: $\frac{\sigma_{eu}}{(x^*)^{\alpha_{eu}}} + \frac{\sigma_{cp}}{(x^*)^{\alpha_{cp}}} = c_1 + c_2$ for optimality. Closed-form solution to **(C1)** is difficult to get when $\alpha_{cp} \neq \alpha_{eu}$. However, by noticing $\frac{\sigma_{eu}}{x^{\alpha_{eu}}} + \frac{\sigma_{cp}}{x^{\alpha_{cp}}}$ is decreasing with x, we can use the bisection algorithm to solve the optimality condition **(C1)**.

Meanwhile, given the optimal rate allocation x^*, the optimal pricing structure for problem **(SPM)** is such that

$$x^* \leq \min\{D_{cp}(h^*_{cp}), D_{eu}(h^*_{eu})\}, \quad (11)$$

$$g^*_{cp} = \sigma_{cp}u_{cp}(x^*) - h^*_{cp}x^*, \quad (12)$$

$$g^*_{eu} = \sigma_{eu}u_{eu}(x^*) - h^*_{eu}x^*. \quad (13)$$

Condition (12) represents that ISP 1's pricing strategy $\{h^*_{cp}, g^*_{cp}\}$ is such that CP is left with zero-utility. Similarly, condition (13) represents that ISP 2's pricing strategy $\{h^*_{eu}, g^*_{eu}\}$ is such that EU is left with zero-utility. Condition (11) requires that the data rate provisioning from CP and the data rate demand from EU should be no smaller than ISPs' optimal rate allocation for problem **(SPM-E)**.

Suppose that we use $x^* = D_{cp}(h^*_{cp})$ and $x^* = D_{eu}(h^*_{eu})$ to determine the optimal usage-prices of ISP 1 and ISP 2, respectively. Then, according to $D_{cp}(h^*_{cp})$ and $D_{eu}(h^*_{eu})$, condition **(C1)** indicates that $h^*_{cp} + h^*_{eu} = c_1 + c_2$, which means that ISPs 1 and 2 use the entire usage-part revenue to cover the marginal costs. Meanwhile, the flat-part revenue constitutes ISPs 1 and 2's optimal social profit $R^*_s = R_s(x^*)$. *Remark:* In the special case of $\alpha_{cp} = \alpha_{eu} = \alpha$, the optimal rate can be expressed as $x^* = (\frac{\sigma_{eu}+\sigma_{cp}}{c_1+c_2})^{\frac{1}{\alpha}}$. By putting x^* into the social profit function $R_s(x)$, we get $R^*_s = R_s(x^*) = (\sigma_{eu} + \sigma_{cp})^{\frac{1}{\alpha}}(\frac{1}{c_1+c_2})^{\frac{1}{\alpha}-1}\frac{\alpha}{1-\alpha}$.

In the following we use $x^* = D_{cp}(h^*_{cp})$ and $x^* = D_{eu}(h^*_{eu})$ to determine the optimal usage-prices for ISP 1 and ISP 2, respectively. Otherwise, part of CP's service rate provisioning will be wasted, or part of EU's rate demand has to be dropped. *Remark:* If $\alpha_{cp} = \alpha_{eu} = \alpha$, we have the optimal usage-price $h^*_i = \frac{\sigma_i}{\sigma_{eu}+\sigma_{cp}}(c_1 + c_2), i = eu, cp$. It indicates that ISPs 1 and 2 proportionally distribute their costs to two sides (i.e., the CP side and EU side) according to the corresponding utility levels. It can be verified that ISP's optimal usage part revenue $R^{usage,*} = x^*(h^*_{eu} + h^*_{cp}) = x^*(c_1 + c_2)$. Meanwhile, we can get the optimal flat-price $g^*_i = \frac{\sigma_i}{\sigma_{ei}+\sigma_{cp}}(\sigma_{eu} + \sigma_{cp})^{\frac{1}{\alpha}}(\frac{1}{c_1+c_2})^{\frac{1}{\alpha}-1}\frac{\alpha}{1-\alpha}, i = eu, cp$. It can be verified that $g^*_{eu} + g^*_{cp} = R^*_s$, i.e., the optimal flat-part revenue constitutes ISPs 1 and 2's optimal social profit.

IV. NONCOOPERATIVE MODEL

We now investigate the noncooperative model in which ISPs fail to collude and thus each ISP selfishly maximizes its own profit. Specifically, we model the strategic interaction between ISPs 1 and 2 as a Stackelberg game. ISP 2, which has the power to determine the transit-price π prior to ISP 1's decision, is the game leader. ISP 1, as a result, is the game follower and determines its traffic rate request in response to ISP 2's decisions. In the Stackelberg game, ISP 2 has the so-called first-move advantage, which means ISP 2 adapts its decisions to maximize its profit by anticipating ISP 1's response. We use the backward induction to derive the noncooperative equilibrium for the Stackelberg game as follows.

First, given ISP 2's transit-price, ISP 1 aims to determine its traffic rate request by solving the following problem

$$\textbf{(ISP1):}\ \max_{x \geq 0}\ \sigma_{cp}u_{cp}(x) - c_1 x - \pi x.$$

Specifically, $\sigma_{cp}u_{cp}(x)$ denotes CP's entire utility extracted by ISP 1. Therefore, ISP 1's rate request can be expressed as $(\frac{\sigma_{cp}}{c_1+\pi})^{\frac{1}{\alpha_{cp}}}$. To be more accurate, in problem **(ISP1)**, ISP 1's traffic rate request cannot exceed the amount provided by ISP 2. We do not consider this constraint here, since the following analysis will show that, in order to maximize its own profit, ISP 2 always adapts its transit-price such that its traffic rate is equal to ISP 1's traffic rate request.

Knowing ISP 1's best-response, ISP 2 determines its traffic rate and transit-price by solving the following problem

$$\textbf{(ISP2):} \quad \max_{x\geq 0, \pi\geq 0} \ \sigma_{eu} u_{eu}(x) - c_2 x + \pi x, \quad \text{subject to: } x \leq \left(\frac{\sigma_{cp}}{c_1+\pi}\right)^{\frac{1}{\alpha_{cp}}}. \tag{14}$$

The optimal solution for problem **(ISP2)** is obtained when constraint (14) is binding strictly, shown as follows. Let (x^e, π^e) denote the optimal solution for problem **(ISP2)**. Suppose that $x^e < (\frac{\sigma_{cp}}{c_1+\pi^e})^{\frac{1}{\alpha_{cp}}}$, then ISP 2 can further increase π^e to increase its profit, thus contradicting with the assumption of the optimality. The binding of constraint (14) means that ISP 2 always adapts its transit-price such that its traffic rate is equal to ISP 1's traffic rate request.

Notice that ISP 2's optimal decision (x^e, π^e) also serves as the noncooperative equilibrium for the Stackelberg game, i.e., ISP 2 determines its traffic rate equal to x^e and its transit-price equal to π^e. As a response to ISP 2's decisions, ISP 1 requires its traffic rate equal to x^e exactly. Let R_s^e denote the corresponding social profit at the noncooperative equilibrium. We quantify the loss in social profit as follows.

Theorem *2:* Assume $\alpha_{cp} = \alpha_{eu} = \alpha$, then $\frac{R_s^e}{R_s^*} = G$, where $G = (1 - \frac{\alpha\sigma_{cp}}{\sigma_{eu}+\sigma_{cp}})^{\frac{1}{\alpha}-1} \frac{\sigma_{eu}+(2-\alpha)\sigma_{cp}}{\sigma_{eu}+\sigma_{cp}}$.

We can further characterize the profit ratio G as follows.

Theorem *3:* Assume $\alpha_{cp} = \alpha_{eu} = \alpha$, then the profit ratio G is increasing with σ_{eu} and is decreasing with σ_{cp}.

Remark: In the Stackelberg model, ISP 2 is the game leader that has the dominant position and determines the transit-price prior to ISP 1. As a result, the utility level of EU (whose access service is provided by ISP 2) can be fully exploited. Therefore, profit ratio G is increasing with σ_{eu}. In contrast, in the Stackelberg model, ISP 1 is the follower which can only optimize its traffic amount in response to ISP 2's decision. As a result, the utility level of CP (whose access service is provided by ISP 1) is poorly exploited. In fact, CP could have produced a larger profit if ISP 2 charges ISP 1 more appropriately. Therefore, the profit ratio G is decreasing with σ_{cp}. Figure 2 shows the decreasing (increasing) property of the profit ratio $G = \frac{R_s^e}{R_s^*}$ with σ_{cp} (σ_{eu}) under different $\alpha_{cp} = \alpha_{eu} = \alpha$.

Theorem *4:* In the special case of $\alpha_{cp} = \alpha_{eu} = \alpha$, the profit ratio G is increasing with α when $0 < \alpha < 1$.

Remark: The noncooperative behaviors of ISP 1 and ISP 2 adversely influence the effective traffic rate from CP to EU, and thus result in a social profit loss compared to the optimal one (i.e., R_s^* corresponding to the optimal traffic rate x^*). Notice that in the noncooperative model, ISP 1 charges CP with $c_1+\pi^e$, which is different from that in the revenue sharing model. However, the social profit loss decreases when CP and EU become more price inelastic. Therefore, ISPs' profit ratio G is increasing with α. Figure 3 shows this property with $\alpha = \alpha_{cp} = \alpha_{eu}$ under different σ_{cp} and σ_{eu}. We also plot the lower bound $\frac{2}{e}$ (i.e., the bottom dash-dot line) for both the case of $\alpha \to 0$ and $\sigma_{cp} \to \infty$ and the case of $\alpha \to 0$ and $\sigma_{eu} \to 0$.

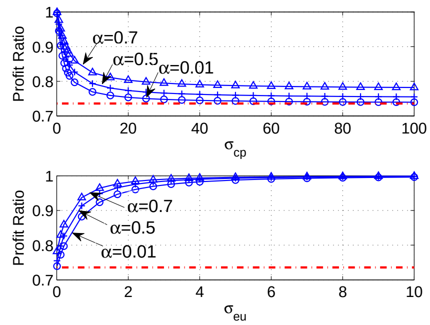

Fig. 2. Performance of $G = \frac{R_s^e}{R_s^*}$. Top subfigure: the ratio G decreases with σ_{cp}. We fix $\sigma_{eu} = 1$ and fix $c_1 = c_2 = 1$; Bottom subfigure: ratio G increases with σ_{eu}. We fix $\sigma_{cp} = 1$. Dash-dot line denotes the value of $\frac{2}{e}$.

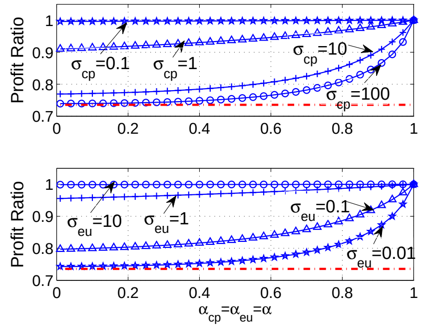

Fig. 3. The profit ratio $G = \frac{R_s^e}{R_s^*}$ increases with consumer's price inelasticity (the price elasticity $\epsilon = \frac{1}{\alpha}$). Top subfigure: We fix $\sigma_{eu} = 1$; Bottom subfigure: We fix $\sigma_{cp} = 0.5$. Dash-dot line denotes the value of $\frac{2}{e}$.

Based on Theorem 4, we can further derive the lower bound of G as $\underline{G} = \lim_{\alpha\to 0} G = (\frac{1}{e})^{\frac{\sigma_{cp}}{\sigma_{eu}+\sigma_{cp}}} \frac{\sigma_{eu}+2\sigma_{cp}}{\sigma_{eu}+\sigma_{cp}}$. Specifically, there exists $\underline{G} \to 1$ when $\sigma_{eu} \to \infty$, and $\underline{G} \to \frac{2}{e}$ when $\sigma_{eu} \to 0$. These results verify the property that G is increasing with σ_{eu}. Meanwhile, there exists $\underline{G} \to \frac{2}{e}$ when $\sigma_{cp} \to \infty$, and $\underline{G} \to 1$ when $\sigma_{cp} \to 0$. These results verify the property that G is decreasing with σ_{cp}.

We also derive the upper bound of G as $\overline{G} = \lim_{\alpha\to 1} G = 1$ i.e., the price inelasticity can mitigate the profit loss.

V. Profit Division Scheme by Asymmetric NBS

The proposed sharing contract provides a mechanism to achieve the optimal social profit. However, each individual ISP's profit still depends on the division factor θ, left undetermined in Section III. In this section, we propose a profit division scheme based on the asymmetric Nash Bargaining Solution (NBS) [5]. In our scenario, the noncooperative equilibrium of the Stackelberg game is considered as the *threat point*. It implies without using the sharing contract, ISP 1 (ISP 2) still achieves the equilibrium profit R_1^e (R_2^e) from the noncooperative game. Let θ^* represent the optimal division factor based on asymmetric NBS. We first quantify the viable range for θ^* such that both ISPs can be simultaneously better off compared to the noncooperative equilibrium.

Specifically, ISP 1's optimal profit can be expressed as (we now treat $G = \frac{R_s^e}{R_s^*}$ as the general profit ratio even if $\alpha_{cp} \neq \alpha_{eu}$): $R_1^* = \theta^* R_s^* = \theta^* \frac{R_s^e}{G} = \theta^* \frac{R_1^e + R_2^e}{G}$. To guarantee $R_1^* \geq R_1^e$, it requires condition **(C2):** $\theta^* \geq \frac{R_1^e}{R_1^e + R_2^e} G$. Similarly, ISP 2's optimal profit can be expressed as $R_2^* = (1-\theta^*) R_s^* = (1-\theta^*) \frac{R_1^e + R_2^e}{G}$. Thus, to guarantee $R_2^* \geq R_2^e$, it requires condition **(C3):** $1 - \frac{R_2^e}{R_1^e + R_2^e} G \geq \theta^*$. Therefore, the viable range for the profit division factor is $\theta^* \in \Theta = \left[\frac{R_1^e}{R_1^e + R_2^e} G, 1 - \frac{R_2^e}{R_1^e + R_2^e} G\right]$. Notice that Θ is nonempty.

Theorem *5:* In the special case of $\alpha_{cp} = \alpha_{eu} = \alpha$, to guarantee that both ISPs can be better off simultaneously, the optimal profit division factor θ^* must satisfy condition **(C4)**:

$$\frac{\sigma_{cp}}{\sigma_{cp} + \sigma_{eu}} \left(1 - \frac{\alpha \sigma_{cp}}{\sigma_{cp} + \sigma_{eu}}\right)^{\frac{1}{\alpha} - 1} \leq \theta^* \leq 1 - \left(1 - \frac{\alpha \sigma_{cp}}{\sigma_{cp} + \sigma_{eu}}\right)^{\frac{1}{\alpha}}$$

Remark: When $\alpha \to 1$, condition **(C4)** becomes $\theta = \frac{\sigma_{cp}}{\sigma_{cp} + \sigma_{eu}}$. It is consistent with the previous result that the profit ratio $G = \frac{R_s^e}{R_s^*} \to 1$ when $\alpha \to 1$. Consequently, the condition for both ISPs to be better off becomes very stringent. Meanwhile, when $\alpha \to 0$, condition **(C4)** becomes $1 - \left(\frac{1}{e}\right)^{\frac{\sigma_{cp}}{\sigma_{cp} + \sigma_{eu}}} \geq \theta \geq \frac{\sigma_{cp}}{\sigma_{cp} + \sigma_{eu}} \left(\frac{1}{e}\right)^{\frac{\sigma_{cp}}{\sigma_{cp} + \sigma_{eu}}}$. It is also consistent with the previous result that $G \to \left(\frac{1}{e}\right)^{\frac{\sigma_{cp}}{\sigma_{eu} + \sigma_{cp}}} \frac{\sigma_{eu} + 2\sigma_{cp}}{\sigma_{eu} + \sigma_{cp}}$ when $\alpha \to 0$. Consequently, the condition for both ISPs can be better off becomes relatively loose.

We now use the asymmetric NBS to determine θ. Mathematically, the Profit Division problem is as follows.

$$\textbf{(PD):} \max_{0 \leq \theta \leq 1} (\theta R_s^* - r_1)^{w_1} ((1-\theta) R_s^* - r_2)^{w_2}$$
$$\text{subject to: } \theta R_s^* - r_1 \geq 0 \text{ and } (1-\theta) R_s^* - r_2 \geq 0,$$

where w_1, w_2 denote ISP 1 and ISP 2's bargaining power respectively. r_1, r_2 denote ISP 1 and ISP 2's minimum profit requirements. As described before, we set $r_1 = R_1^e$ and $r_2 = R_2^e$. The optimal solution for problem (**PD**) can be uniquely determined by $R_i^* = r_i + (R_s^* - \sum_{i=1,2} r_i) \frac{w_i}{\sum_{i=1,2} w_i}$, $i = 1, 2$. The optimal profit division reveals the rationale behind the asymmetric NBS, i.e., as long as the *threat point* is met, the additional social profit from ISPs' collusion will be fairly distributed according to their bargaining power.

Meanwhile, the optimal profit division factor can be expressed as $\theta^* = (1 - \frac{r_1 + r_2}{R_s^*}) \frac{w_1}{w_1 + w_2} + \frac{r_1}{R_s^*}$. It can be verified that $0 \leq \theta^* \leq 1$ as long as $r_1 + r_2 \leq R_s^*$.

Remark: If $\alpha_{cp} = \alpha_{eu} = \alpha$, the optimal division factor θ^* has to satisfy condition **(C4)**. Figure 4 illustrates this point. Specifically, increasing w_1 can increase ISP1 's profit until ISP 2's profit reaches its minimum requirement. Correspondingly, θ^* increases until it hits the upper bound of condition **(C4)**.

VI. Conclusion

We study the revenue sharing and rate allocation among ISPs that jointly deliver traffic from CPs to EUs. Noncooperative inter-pricing inevitably incurs a loss in ISPs' social profit. Our results quantify that the loss decreases (increases) with the utility level of customer whose connectivity is provided by the leader-ISP (the follower-ISP). Meanwhile, customer's price inelasticity can mitigate the profit loss. To recover the loss in social profit, we propose a revenue sharing contract for ISPs to collude. By appropriately structuring the transit-price, the sharing contract coordinates ISPs' objectives such that they aim to maximize the social profit self-incentively. Based on the sharing contract, we further quantify the viable range for the profit division factor such that each ISP can be better off compared to the noncooperative equilibrium. Finally, we use the asymmetric NBS to determine the optimal profit division factor. In [10] we develop a distributed procedure to implement the bargaining process without revealing ISPs' private information. We also consider the limitation of the revenue sharing contract, and invoke the "trigger strategy" in game theory to overcome the limitation.

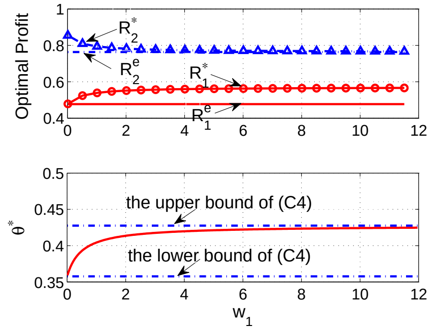

Fig. 4. Profit division scheme with asymmetric NBS. We set $\sigma_{cp} = \sigma_{eu} = 1$. $\alpha_{cp} = \alpha_{eu} = 0.4$. $c_1 = c_2 = 1$. We fix $w_2 = 0.5$. Top subfigure: R_1^* and R_2^* with different w_1. Bottom subfigure: θ^* with different w_1.

References

[1] P. Hande, M. Chiang, R. Calderband, J.S. Zhang, "Network Pricing and Rate Allocation with Content Provider Participation," *in Proc. of INFOCOM2009*

[2] G. Huston, "Interconnection, Peering, and Settlement", available at http://au.net/papers/1999-6-peer/peering.pdf

[3] S. Shakkottai, R. Srikant, "Economics of Network Pricing with Multiple ISPs", *IEEE/ACM Transactions on Networking*, vol 14, no 6, pp. 1233-1245, Dec. 2006

[4] G.P. Cachon, M.A., Lariviere, "Supply Chain Coordination with Revenue-Sharing Contracts: Strengths and Limitations," *Management Science*, vol. 51, no. 1, pp. 30-44, Jan. 2005

[5] K. Binmore, A. Rubinstein, Asher Wolinsky, "The Nash Bargaining Solution in Economic Modelling", *RAND Journal of Economics*, vol. 17, no. 2, pp. 176-188, 1986

[6] S.C.M. Lee, J.W.J. Jiang, D.M. Chiu, J.C.S. Lui, "Interaction of ISPs: Distributed Resource Allocation and Revenue Maximization", *IEEE Transactions on Parallel and Distributed Systems*, vol. 19, no. 2, pp. 204-218, Feb. 2008

[7] J. H. Wang, D. M. Chiu, J. C.S. Lui, "A Game-theoretic Analysis of the Implications of Overlay Network Traffic on ISP Peering", *Computer Networks*, vol. 52, no. 15, pp. 2961-2974, Oct. 2008.

[8] R.T.B. Ma, D.M. Chiu, J.C.S. Lui, V. Misra, D. Rubenstein, "Interconnecting Eyeballs to Content: A Shapley Vlaue Perspective on ISP Peering and Settlement", *in Proc. of NetEcon'08*

[9] L. He, J. Walrand, "Pricing and Revenue Sharing Strategies for Internet Service Provider", *IEEE Journal of Selected Areas on Communications*, vol. 24, no. 5, pp. 942-951, May 2006.

[10] Y. Wu, H. Kim, P.H. Hande, M. Chiang, D.H.K. Tsang, "Revenue Sharing among ISPs in Two-sided Markets", available at http://eez058.ee.ust.hk/publication/ISPrevenuesharing.pdf